A Simple and Efficient Error-Diffusion Algorithm

Victor Ostromoukhov
Université de Montréal

http://www.iro.umontreal.ca/~ostrom

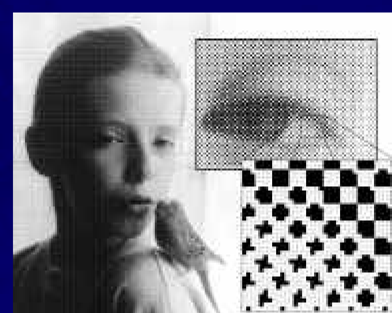
Motivations

- Error-Diffusion is Important Visualization
- No Satisfactory Solution Exists Today
- A Simple and Efficient Solution is Possible

Outline

- Introduction
- Problem Statement
- Our Variable-Coefficient Error-Diffusion Algorithm
- Results
- Conclusions and Future Work

What is Halftoning?



Where Do We Need Halftoning?

Displays

PDAs, Mobile Phones

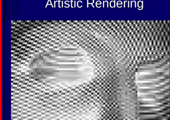
Games

Target Devices for Our Algorithm: Low Resolution, Where Individual Pixels Are Visible

Printers

Network-Based Imaging

Artistic Rendering

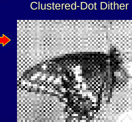


Halftoning: Basic Classification

Continuous-Tone Original

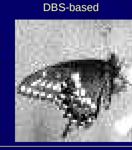
Monte-Carlo

Clustered-Dot Dither

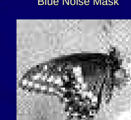


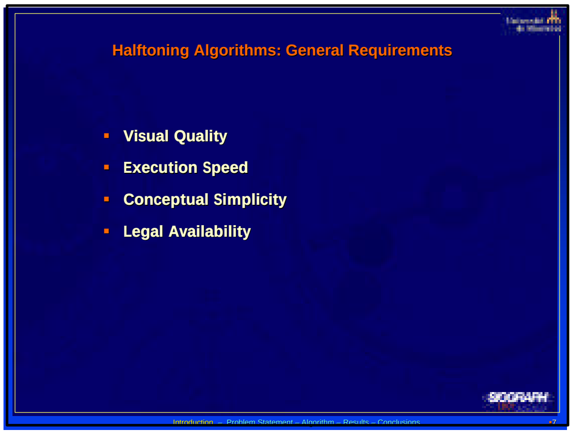
Error-Diffusion

DBS-based



Blue Noise Mask




Halftoning Algorithms: General Requirements
Visual Quality
Execution Speed
Conceptual Simplicity
Legal Availability
Introduction – Problem Statement – Algorithm – Results – Conclusions
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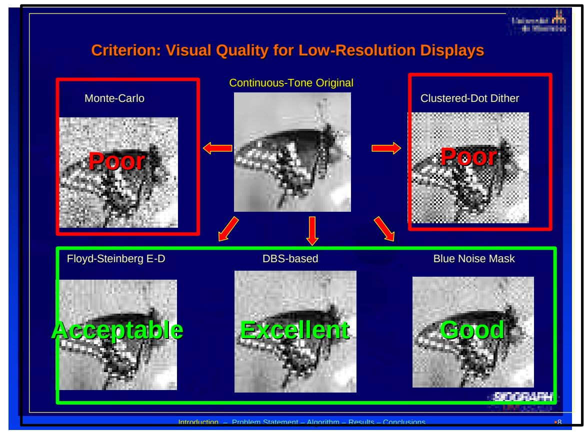
Criterion: Visual Quality for Low-Resolution Displays
Monte-Carlo
Poor
Continuous-Tone Original
Clustered-Dot Dither
Poor
Floyd-Steinberg E-D
Acceptable
DBS-based
Excellent
Blue Noise Mask
Good
Introduction – Problem Statement – Algorithm – Results – Conclusions
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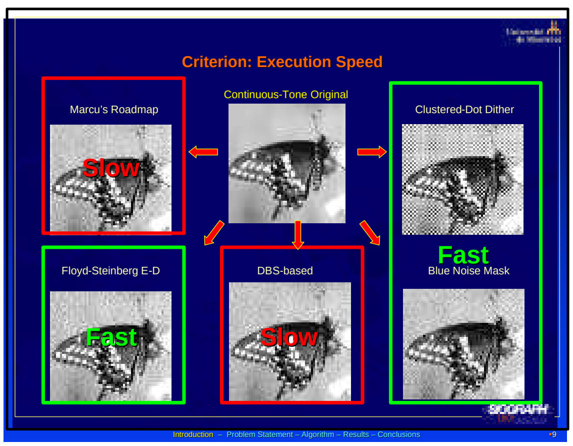
Criterion: Execution Speed
Marcu's Roadmap
Slow
Continuous-Tone Original
Clustered-Dot Dither
Fast
Blue Noise Mask
Floyd-Steinberg E-D
Fast
DBS-based
Slow
Introduction – Problem Statement – Algorithm – Results – Conclusions
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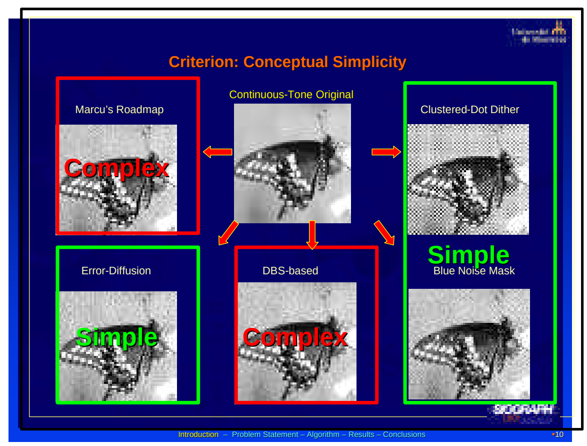
Criterion: Conceptual Simplicity
Marcu's Roadmap
Complex
Continuous-Tone Original
Clustered-Dot Dither
Simple
Blue Noise Mask
Error-Diffusion
Simple
DBS-based
Complex
Introduction – Problem Statement – Algorithm – Results – Conclusions
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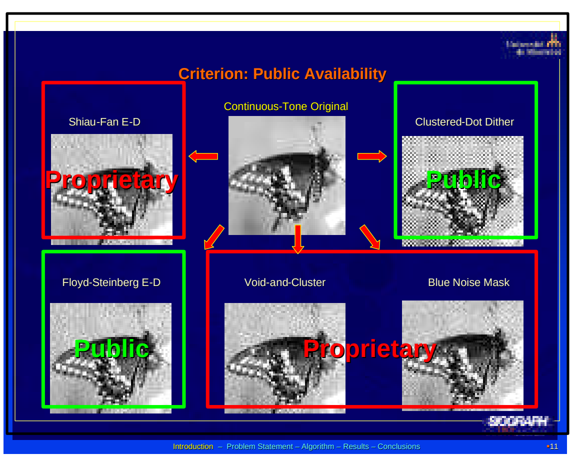
Criterion: Public Availability
Shiau-Fan E-D
Proprietary
Continuous-Tone Original
Clustered-Dot Dither
Public
Floyd-Steinberg E-D
Public
Void-and-Cluster
Blue Noise Mask
Proprietary
Introduction – Problem Statement – Algorithm – Results – Conclusions
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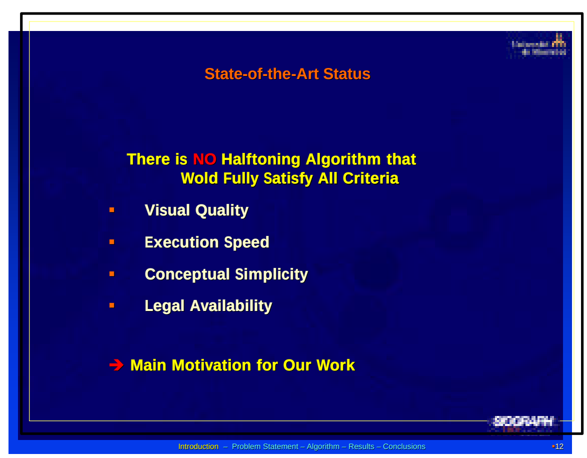
State-of-the-Art Status
There is NO Halftoning Algorithm that
Wold Fully Satisfy All Criteria
Visual Quality
Execution Speed
Conceptual Simplicity
Legal Availability
→ Main Motivation for Our Work
Introduction – Problem Statement – Algorithm – Results – Conclusions
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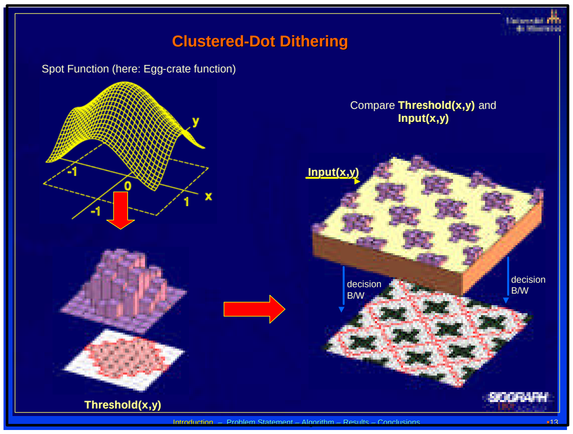
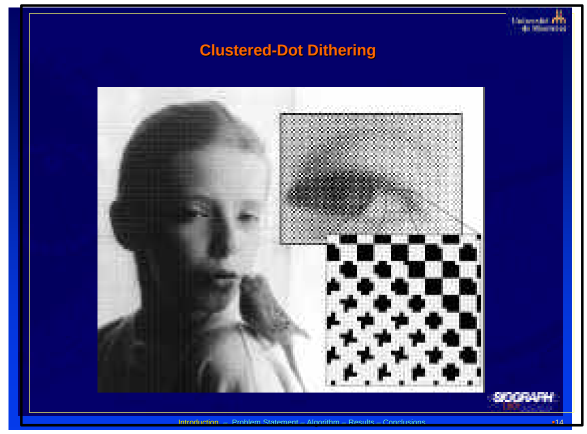

Clustered-Dot Dithering
Spot Function (here: Egg-crate function)
Compare Threshold(x,y) and Input(x,y)
Input(x,y)
decision B/W
decision B/W
Threshold(x,y)
SIGGRAPH
Introduction – Problem Statement – Algorithm – Results – Conclusions
•13


Clustered-Dot Dithering
SIGGRAPH
Introduction – Problem Statement – Algorithm – Results – Conclusions
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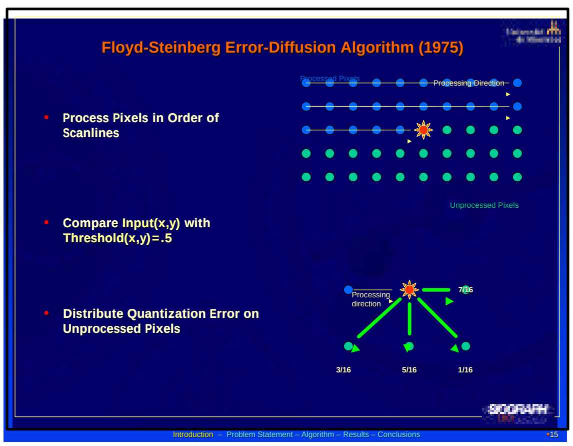
Floyd-Steinberg Error-Diffusion Algorithm (1975)
• Process Pixels in Order of Scanlines
• Compare Input(x,y) with Threshold(x,y)=.5
• Distribute Quantization Error on Unprocessed Pixels
Processed Pixels
Processing Direction
Unprocessed Pixels
Processing direction
7/16
3/16
5/16
1/16
SIGGRAPH
Introduction – Problem Statement – Algorithm – Results – Conclusions
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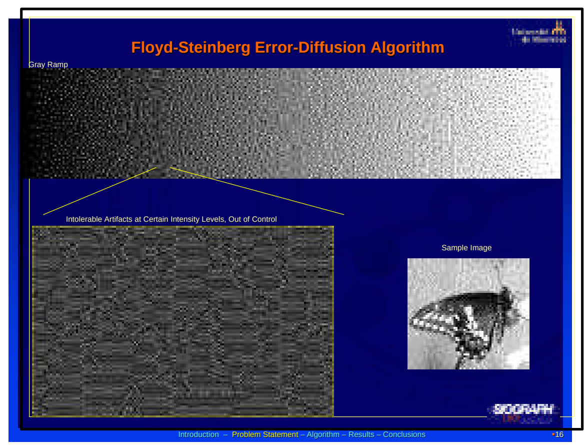
Floyd-Steinberg Error-Diffusion Algorithm
Gray Ramp
Intolerable Artifacts at Certain Intensity Levels, Out of Control
Sample Image
SIGGRAPH
Introduction – Problem Statement – Algorithm – Results – Conclusions
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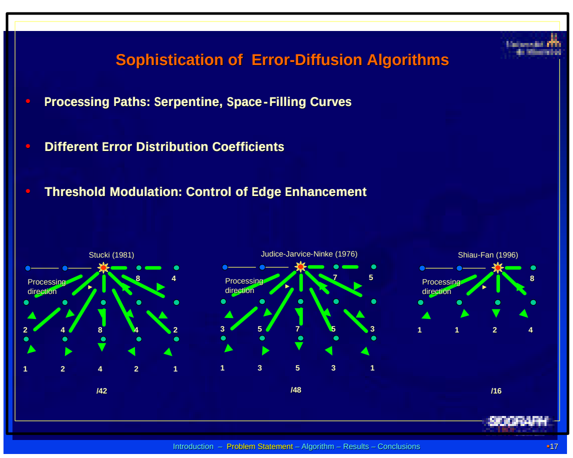
Sophistication of Error-Diffusion Algorithms
• Processing Paths: Serpentine, Space-Filling Curves
• Different Error Distribution Coefficients
• Threshold Modulation: Control of Edge Enhancement
Stucki (1981)
Judice-Jarvice-Ninke (1976)
Shiau-Fan (1996)
Processing direction
/42
/48
/16
SIGGRAPH
Introduction – Problem Statement – Algorithm – Results – Conclusions
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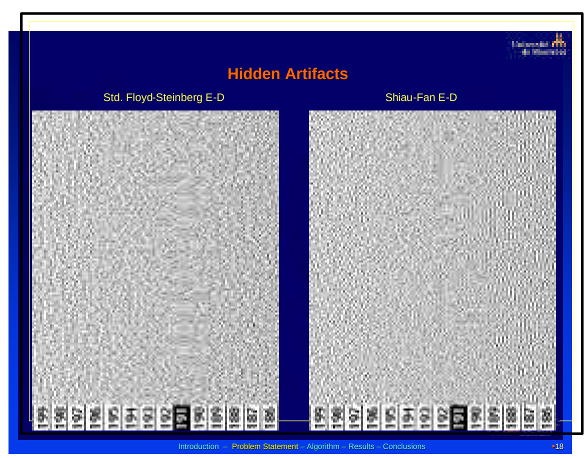
Hidden Artifacts
Std. Floyd-Steinberg E-D
Shiau-Fan E-D
Introduction – Problem Statement – Algorithm – Results – Conclusions
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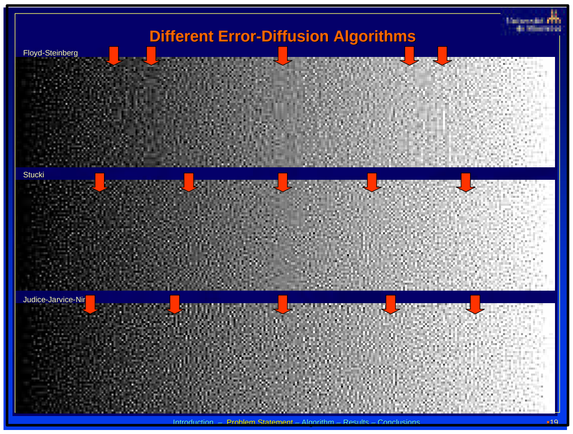
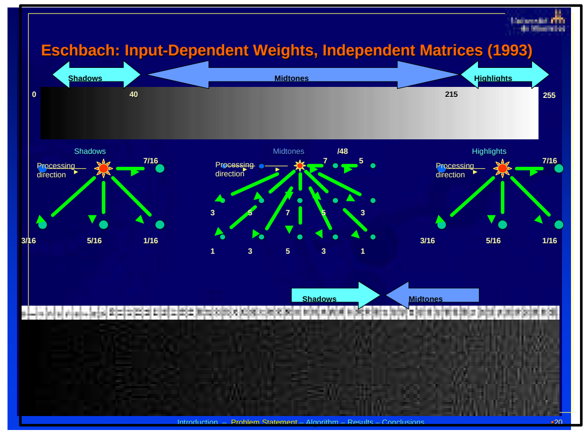
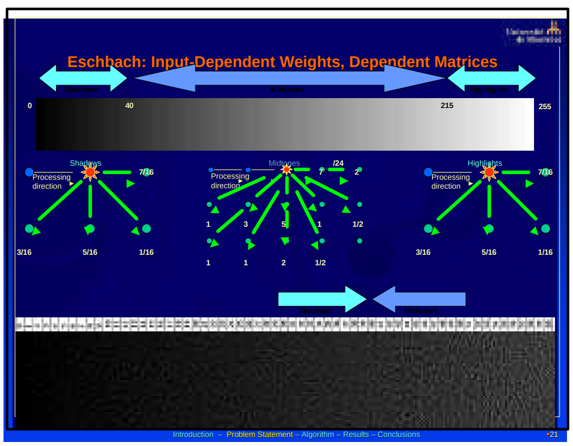






Eschbach: Input-Dependent Weights

Conclusions

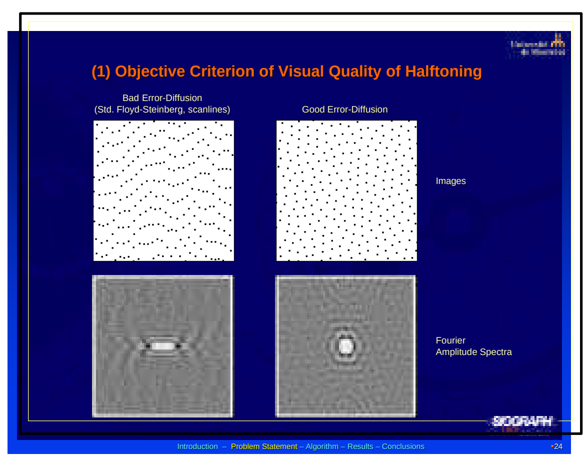
- Visual Artifacts can be improved Using Input-Dependent Weights
- Correlation Between Matrices May Reduce Banding Effect

Weak Points of the Method

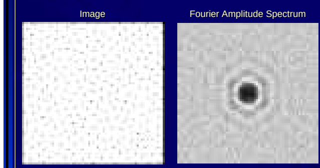
- No Objective Criterion on Input-Dependent Weights
- No Mechanism Proposed To Insure Correlation Between Weights

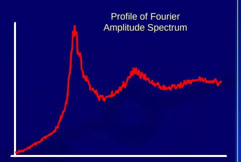
Our Contribution

1. Provide Objective Criterion on Input-Dependent Weights
2. Provide Mechanism To Insure Correlation Between Weights
3. Implement an Error-Diffusion Algorithm
   - Conceptually Simple
   - Computationally Efficient
   - Almost Artifact-Free
   - Publicly Available

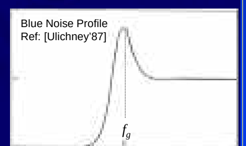


(1) Objective Criterion of Visual Quality of Halftoning

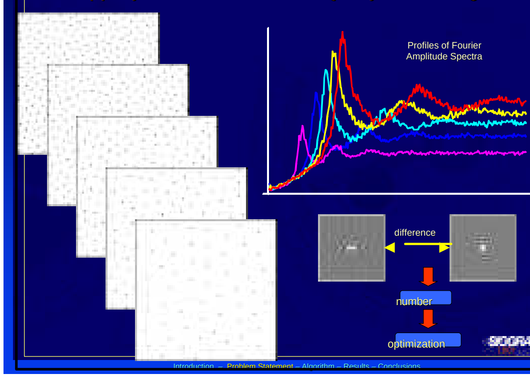




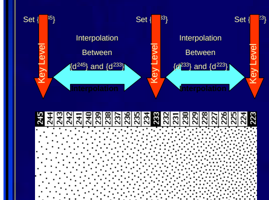
- **Radial Symmetry**
- **Characteristic Blue Noise Profile**



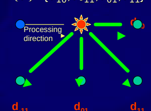
(1) Objective Criterion of Visual Quality of Halftoning



(2) Mechanism of Correlation Between E-D Weights



For each intensity level i,
The set $\{d^i\}$ is defined as
$\{d^i\}=\{d_{10},d_{-11},d_{01},d_{11}\}$



Our Error-Diffusion Algorithm

Assumptions:

A. E-D Algorithm is good when its Fourier Amplitude Spectrum is close to "Blue Noise"

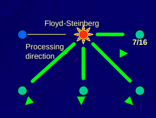
B. Smooth behavior of E-D algorithm when coefficients $\{d^i\}$ vary smoothly across the dynamic range of intensity levels $\{i\}$

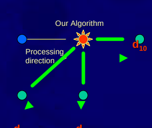
C. Intensity levels 0, **1/4, 1/3, 1/2, 2/3, 3/4, 1** are potentially problematic: they may be source of artifacts

D. Sets $\{d^i\}$ and $\{d^{255-i}\}$ generate almost identical artifacts structures, with white and black inverted

Our Error-Diffusion Algorithm

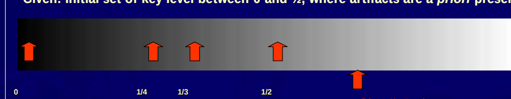
- **Simplified Set of Distribution Coefficients** $d^i=\{d_{10},d_{-11},d_{01}\}$ for Each Intensity Level i
  - **Faster**
  - **Easier to Optimize**
  - **Sufficient for Achieving Good Quality**





Our Error-Diffusion Algorithm: Finding $\{d_{10},d_{-11},d_{01}\}$

Given: Initial set of key level between 0 and ½, where artifacts are *a priori* present



- Step I
  For each key level, find $d^{key}=\{d_{10},d_{-11},d_{01}\}$ that approaches "Blue Noise", by minimizing the difference between given Fourier Amplitude Spectrum and Ideal "Blue Noise Profile"

Our Error-Diffusion Algorithm: Finding $\{d_{10},d_{-11},d_{01}\}$

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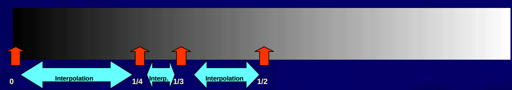

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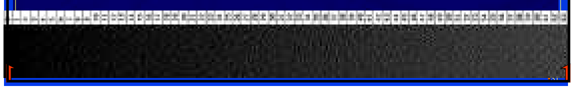
- Step II
  Interpolate between key level.
  Visually check for artifacts in-between.



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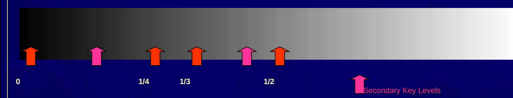

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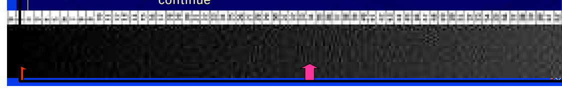
- Step II
  Interpolate between key level.
  Visually check for artifacts in-between.
  If not OK,
  define new key levels and jump to Step I
  otherwise:
  continue



Our Error-Diffusion Algorithm: Finding $\{d_{10},d_{-11},d_{01}\}$

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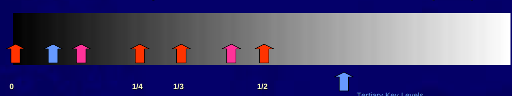

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Our Error-Diffusion Algorithm: Finding $\{d_{10},d_{-11},d_{01}\}$

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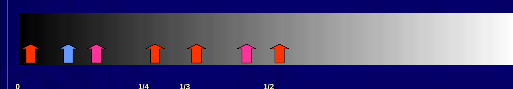

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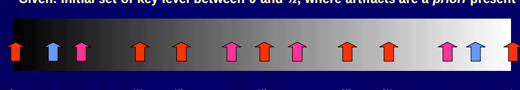
- Step II
  Interpolate between key level.
  Visually check for artifacts in-between.
  If not OK,
  define new key levels and jump to Step I
  otherwise:
  continue
- Step III
  Extend Solution Symmetrically, about ½

Our Error-Diffusion Algorithm: Finding $\{d_{10},d_{-11},d_{01}\}$

Given: Initial set of key level between 0 and ½, where artifacts are *a priori* present



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  Visually check for artifacts in-between.
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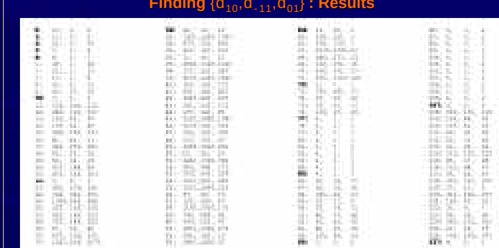
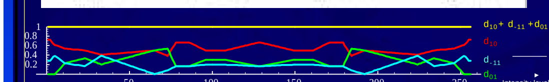
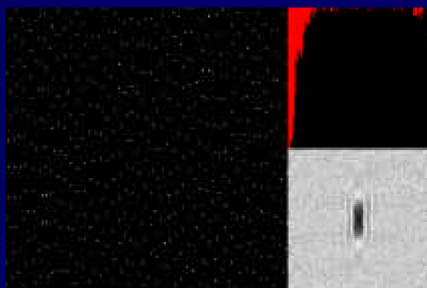
  otherwise:
  continue
- Step III
  Extend Solution Symmetrically, about ½

Finding $\{d_{10},d_{-11},d_{01}\}$: Results



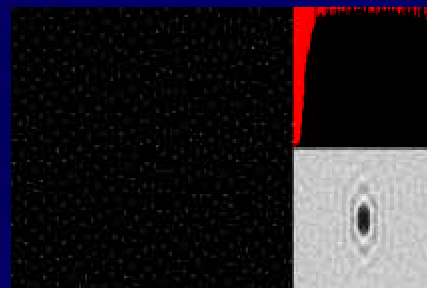


Step I, Iteration # 1



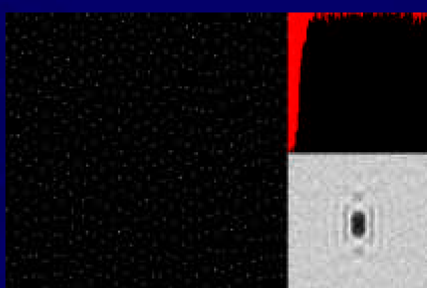
d_{10} = 0.333333
d_{-11} = 0.333333
d_{01} = 0.333333

Step I, Iteration # 3



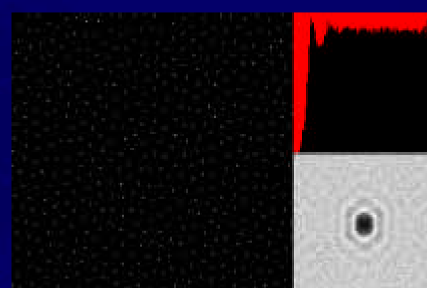
d_{10} = 0.445736
d_{-11} = 0.306202
d_{01} = 0.248062

Step I, Iteration # 7



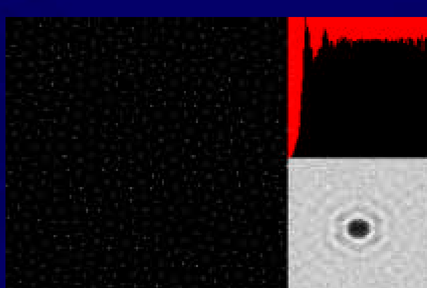
d_{10} = 0.527778
d_{-11} = 0.444445
d_{01} = 0.0277776

Step I, Iteration # 12



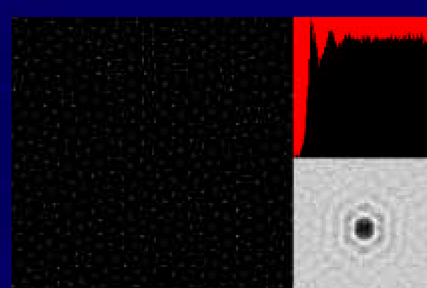
d_{10} = 0.637344
d_{-11} = 0.186967
d_{01} = 0.175689

Step I, Iteration # 15



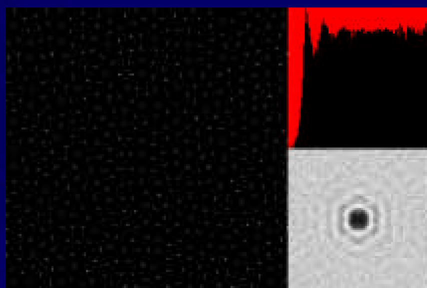
d_{10} = 0.630038
d_{-11} = 0.218744
d_{01} = 0.151219

Step I, Iteration # 20



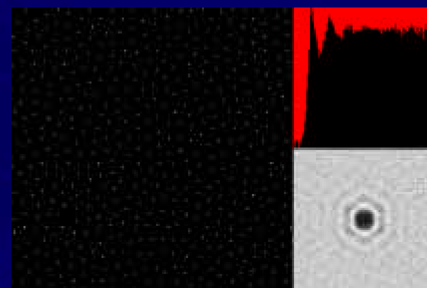
d_{10} = 0.673123
d_{-11} = 0.114484
d_{01} = 0.212393

Step I, Iteration # 33



d_{10} = 0.664381
d_{-11} = 0.185621
d_{01} = 0.149998

Step I, Iteration # 41



d_{10} = 0.655903
d_{-11} = 0.182528
d_{01} = 0.16157

Results

Top: Std. Floyd-Steinberg E-D
Middle: Our Method
Bottom: Shiau-Fan E-D

Results

Top: Std. Floyd-Steinberg E-D
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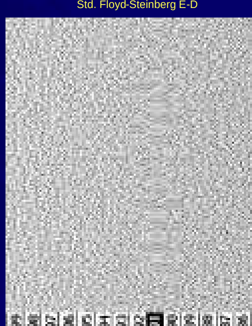
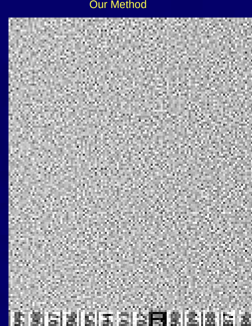
Results

Std. Floyd-Steinberg E-D



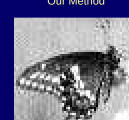
Our Method



Results

Monte-Carlo

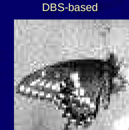
Our Method



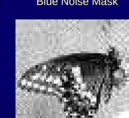
Clustered-Dot Dither

Floyd-Steinberg E-D

DBS-based



Blue Noise Mask



Conclusions

- High-speed Error-Diffusion algorithm
- Good visual quality
- Conceptually simple
- Publicly available on

http://www.iro.umontreal.ca/~ostrom/varcoeffED

- Results of further development will be placed at the same address:

http://www.iro.umontreal.ca/~ostrom/varcoeffED

Future Work

- Taxonomy of Artifacts in E-D
  - Different Artifacts need Different Processing
  - Current Work by P-M Jodoin: Case around ½
- Better Cost Function (Automatic Quality Evaluation)
  - Pathetic Cases (Blue Noise Criterion Does Not Work)
  - Detection of Local Structures (Wavelets, Gabor Functions etc.)
- Systematic Study of Parametric Space Dimensionality
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  - Coherence Through Multiple Depths
  - Temporal Coherence with E-D

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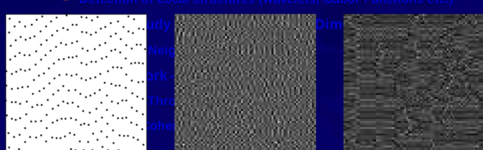
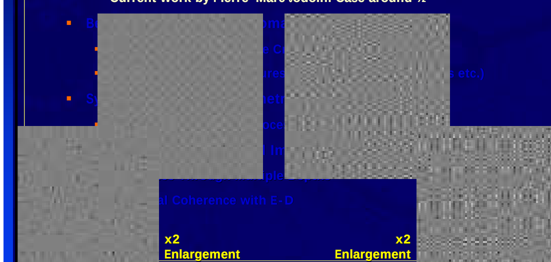


Future Work

- Current Work by Pierre-Marc Jodoin: Case around ½



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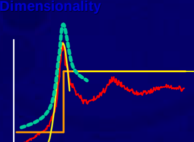
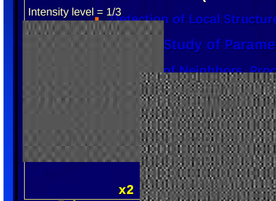
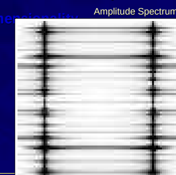


Future Work

- Pathetic Cases (Blue Noise Criterion Does Not Work)





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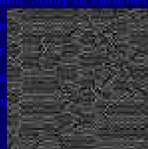

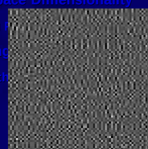

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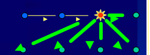

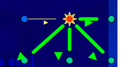

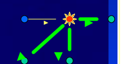

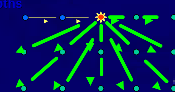

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Thanks

- Bob Ulichney, Jan Allebach, Gabriel Marcu, Reiner Eschbach, Luiz Velho, and Kevin Parker for insightful discussions and for providing sample images of their techniques
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Thank You