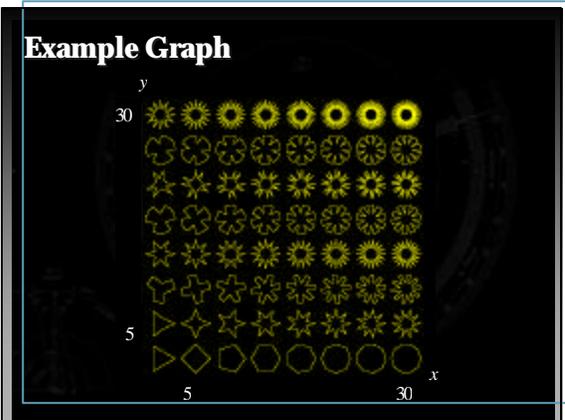
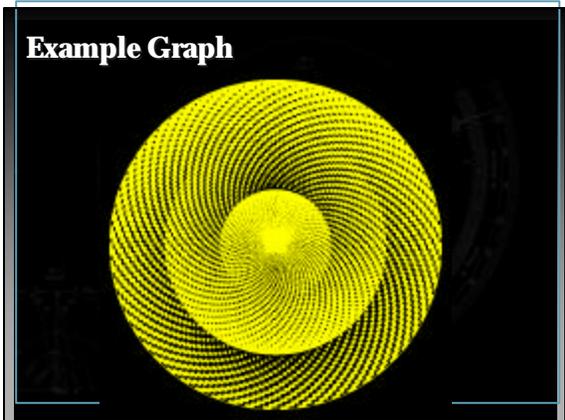
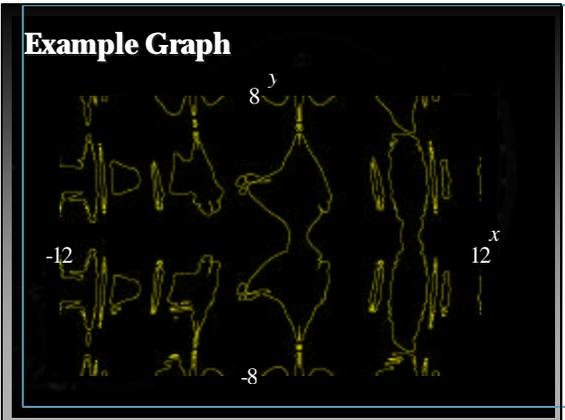
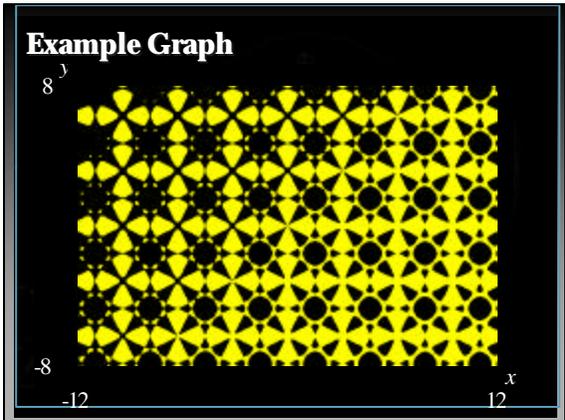
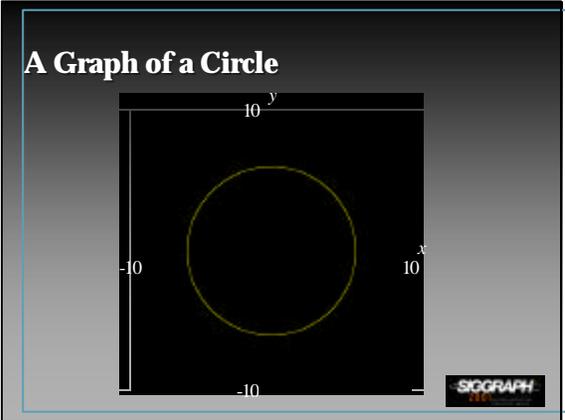
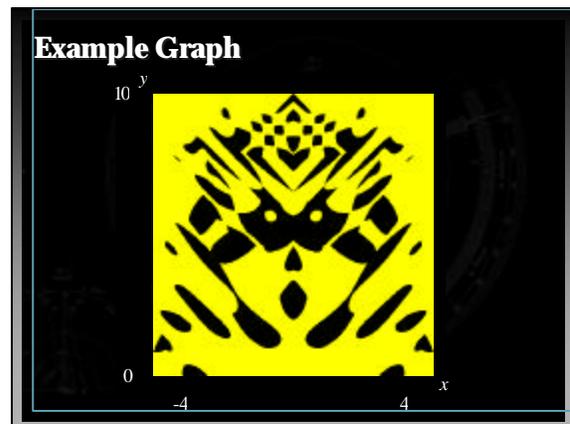
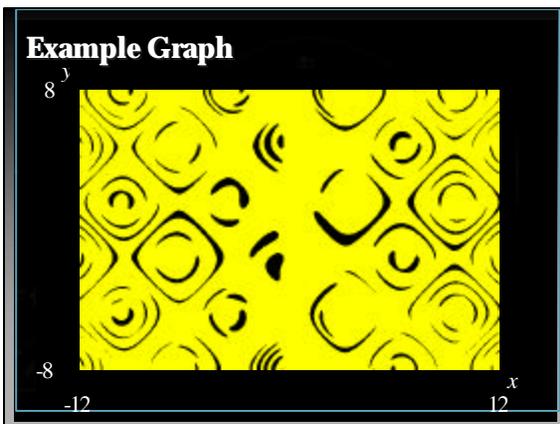
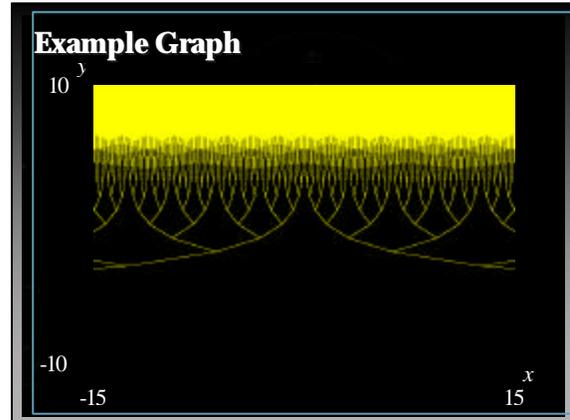
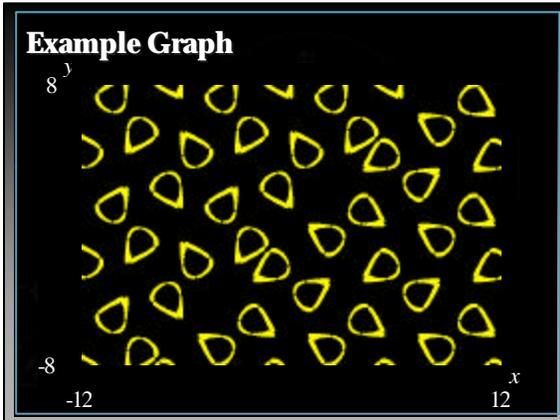


Reliable Two-Dimensional Graphing Methods
for Mathematical Formulae with Two Free Variables

Jeff Tupper
Dynamic Graphics Project
University of Toronto

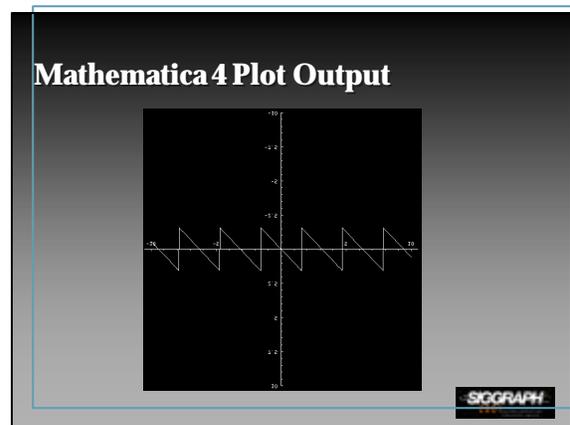


Isn't this already solved?

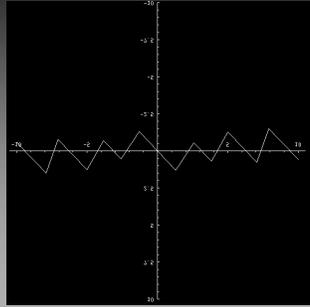
There are many utilities for doing this:

- **Computer Algebra Systems**
 - Mathematica, Maple, ...
- **Graphing Calculators**
 - Hewlett-Packard, Texas Instruments, ...
- **Graphing Software**
 - Curvus Pro, IAsolve, GraphingCalculator, ...

SIGGRAPH

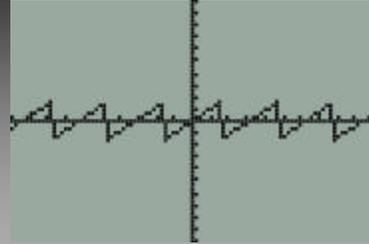


Mathematica 4 ImplicitPlot Output



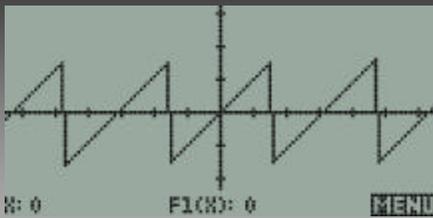
SIGGRAPH

Texas Instruments TI-83 Plus



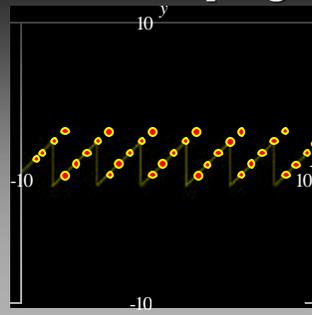
SIGGRAPH

Hewlett-Packard HP 39G



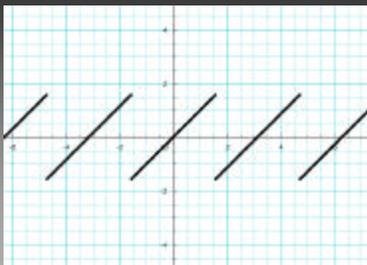
SIGGRAPH

Connect-The-Dots Graphing



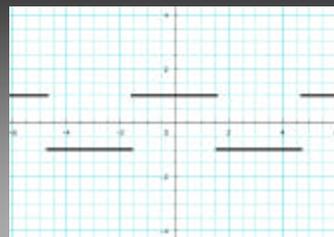
SIGGRAPH

Curvus Pro 3.0.1



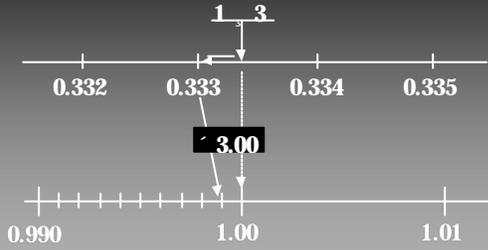
SIGGRAPH

Curvus Pro 3.0.1



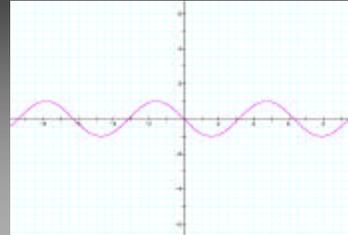
SIGGRAPH

Floating-Point Arithmetic



SIGGRAPH

Graphing Calculator 3.0.1 [Avitzur]



SIGGRAPH

Connect-the-Dots Graphing

Problems:

- Not all dots should be connected
- Dots may be far from the curve

SIGGRAPH

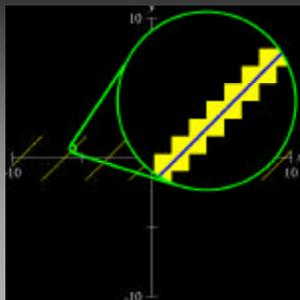
Connect-the-Dots Graphing

Fundamental Problem:

- We haven't defined the graph's semantics

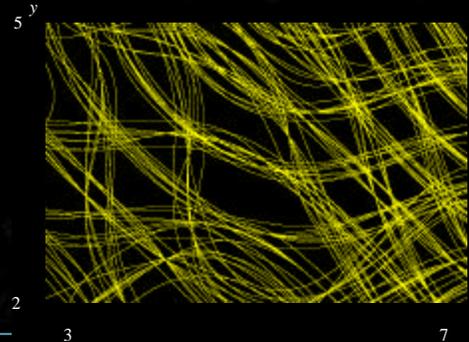
SIGGRAPH

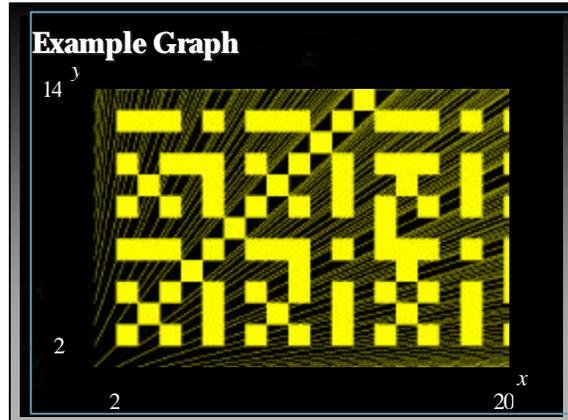
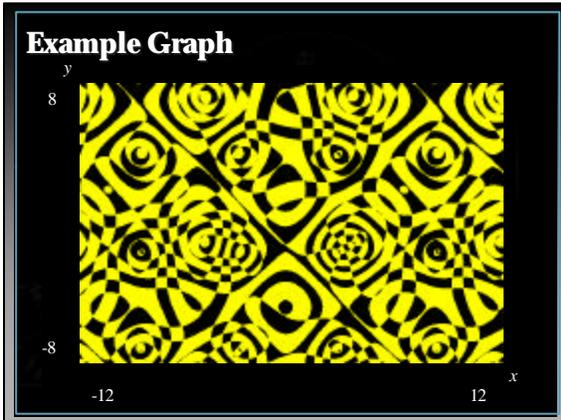
Graph Semantics



SIGGRAPH

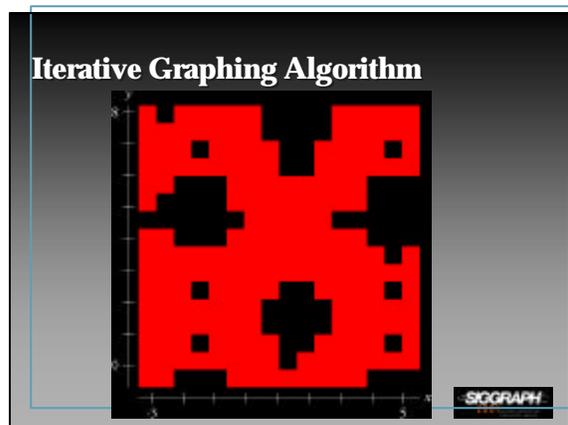
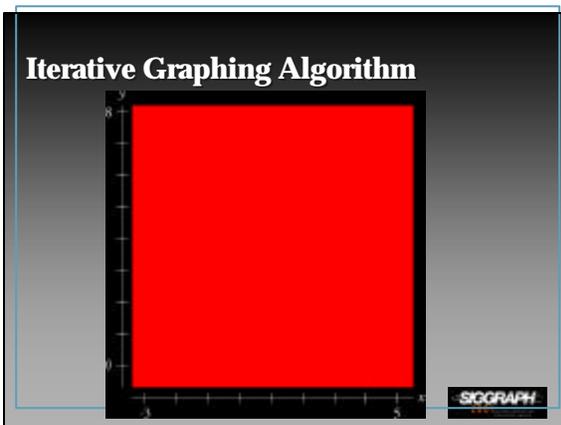
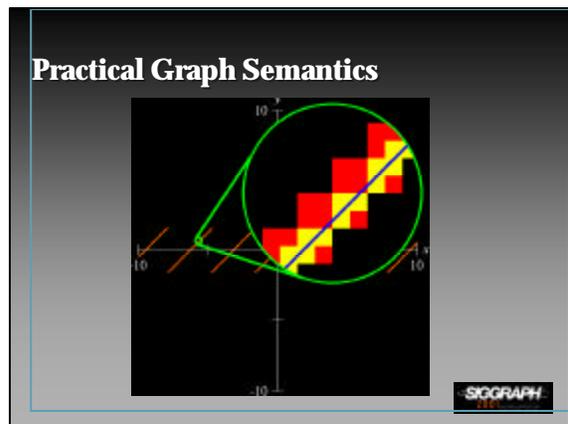
Example Graph

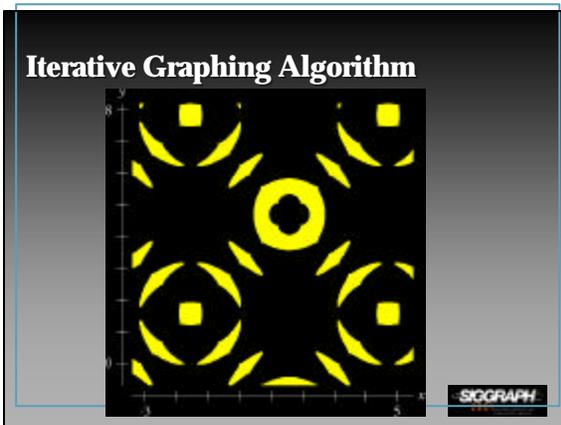
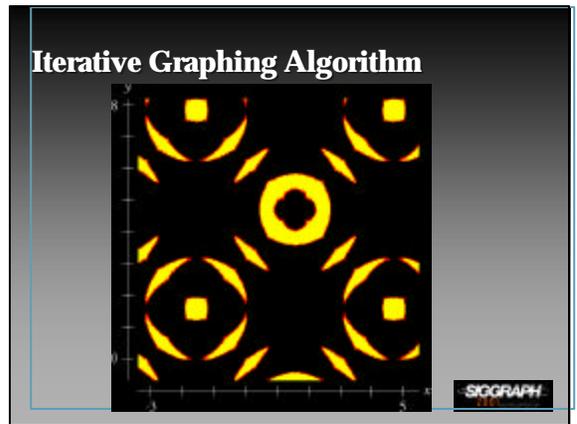
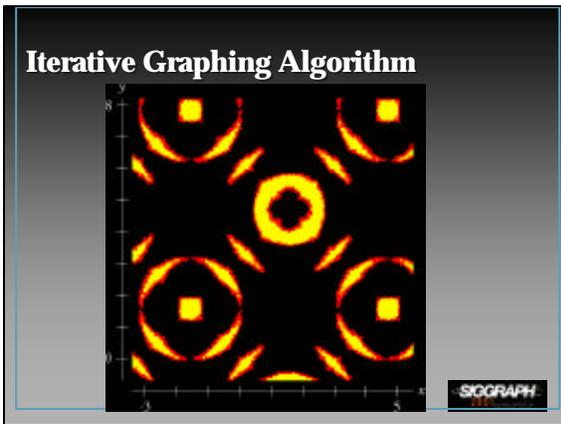
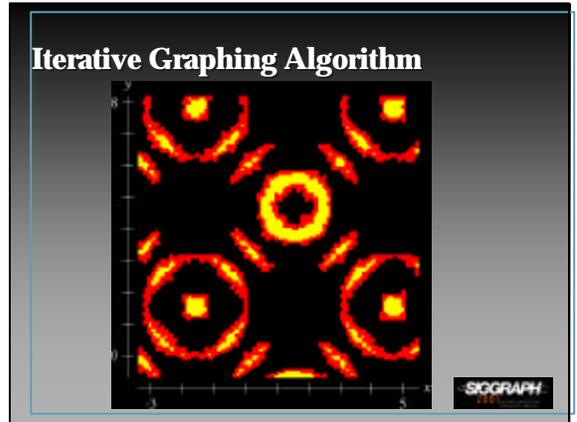
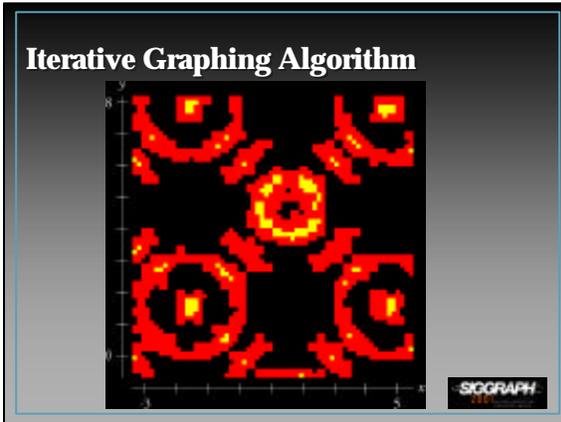




Unfortunate Reality

- This naïve goal is impossible since graphing, as formalized, is not computable





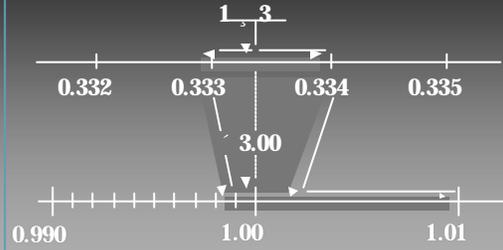
- ### Reliable Graphing
- We now have a well-defined problem
 - But how do we evaluate formulae?
- SIGGRAPH

Formula Evaluation

- Use interval arithmetic to evaluate formulae
- Interval arithmetic provides guaranteed bounds on accuracy

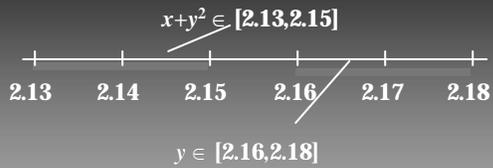
SIGGRAPH

Interval Arithmetic



SIGGRAPH

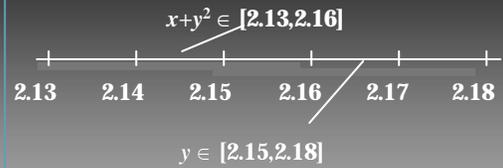
Interval Comparisons



Is $x+y^2 < y$? Yes.

SIGGRAPH

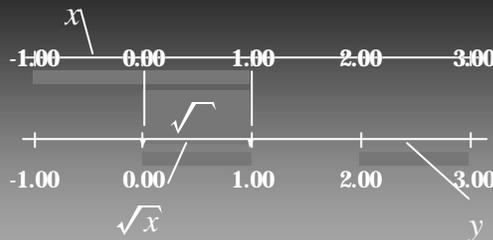
Interval Comparisons



Is $x+y^2 < y$? Maybe.

SIGGRAPH

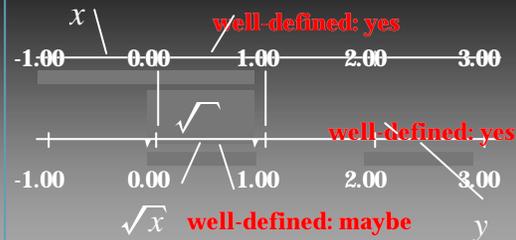
Domain Tracking



Is $\sqrt{x} < y$? Yes.

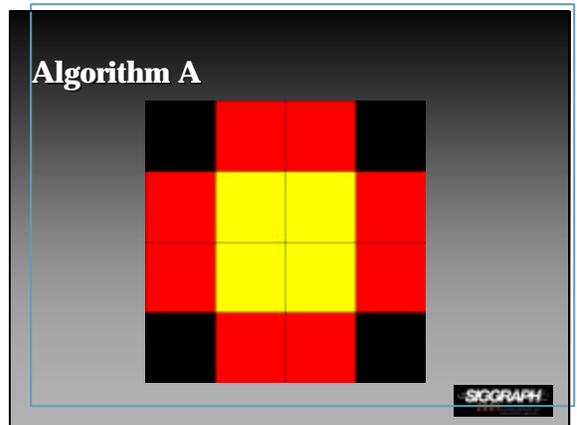
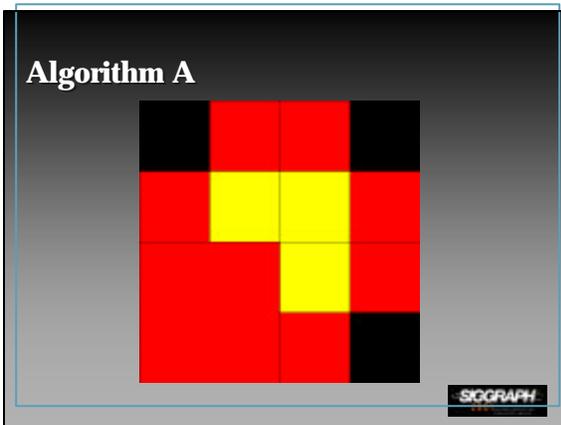
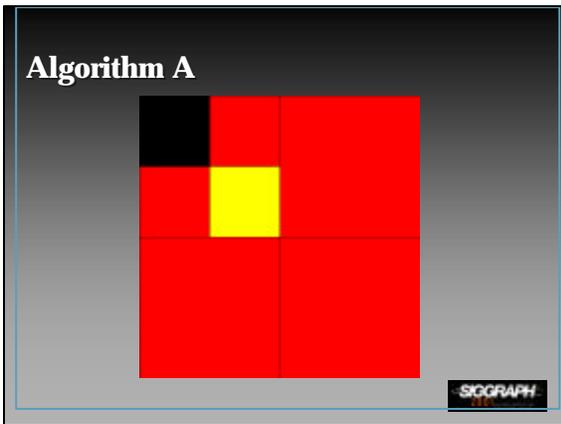
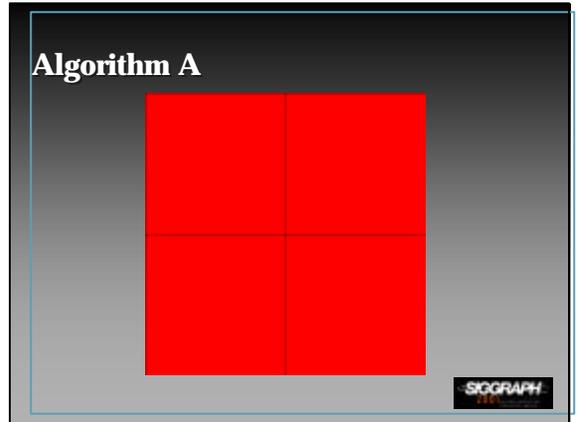
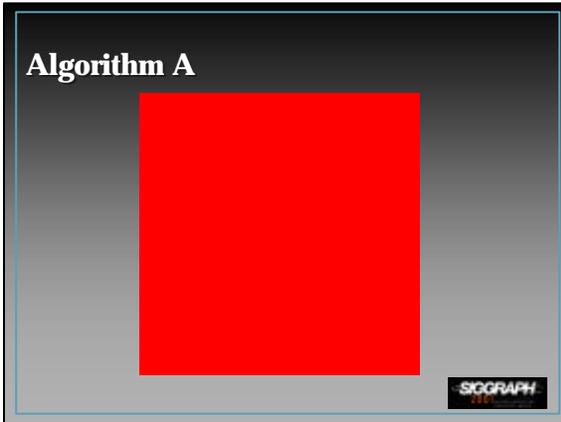
SIGGRAPH

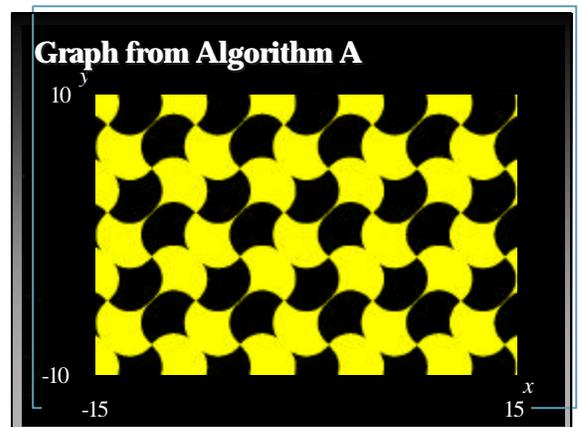
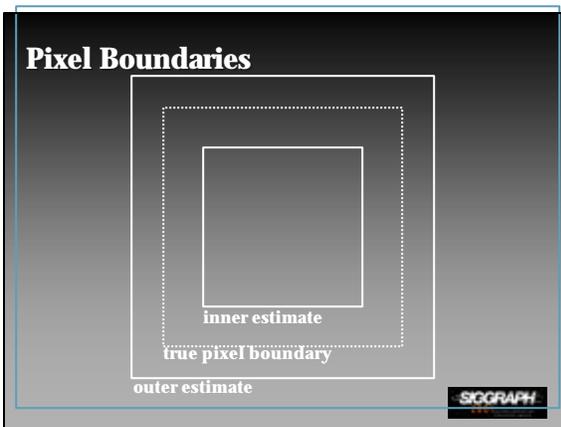
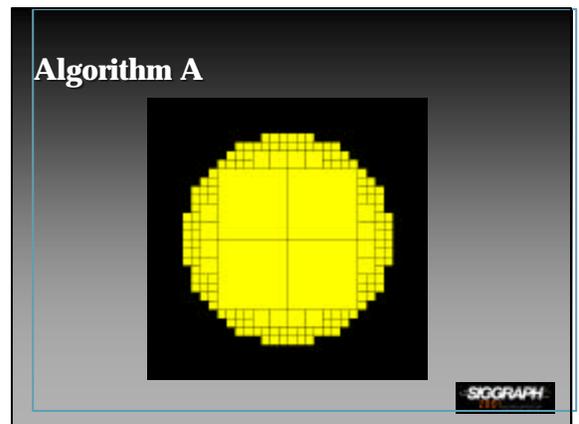
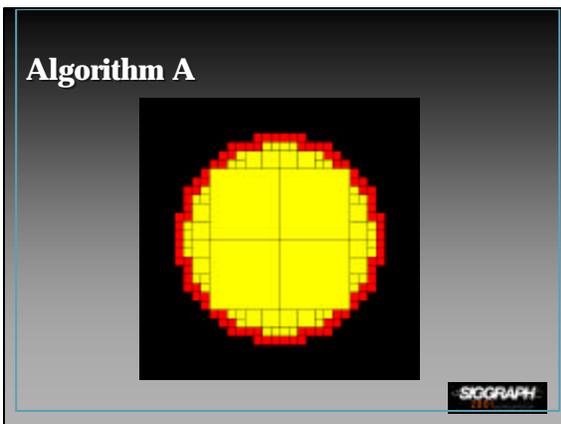
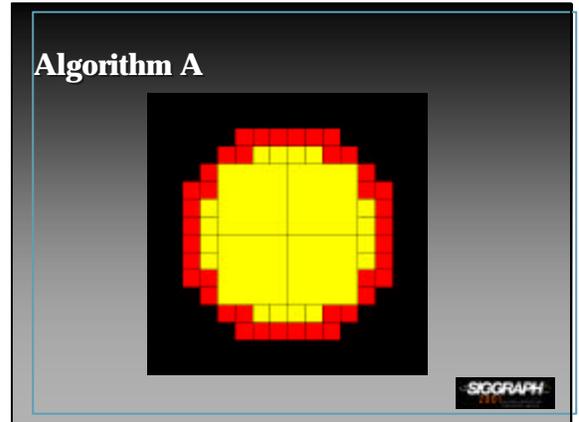
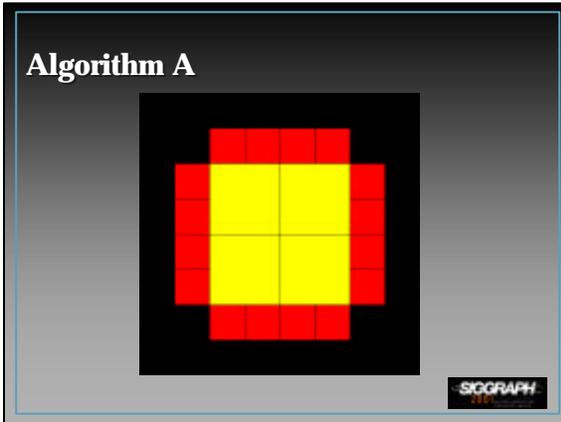
Domain Tracking

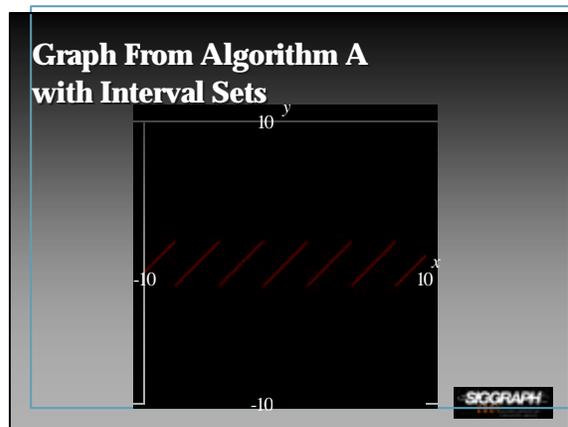
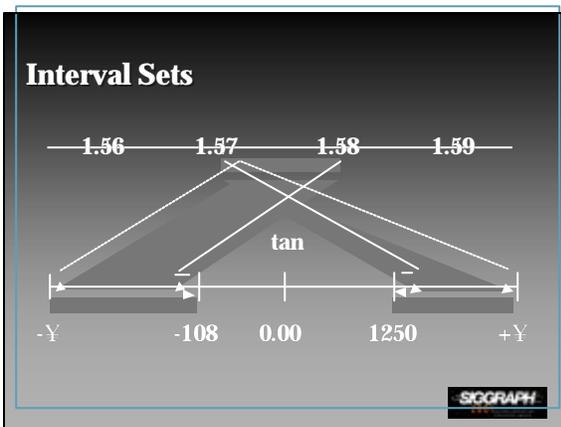
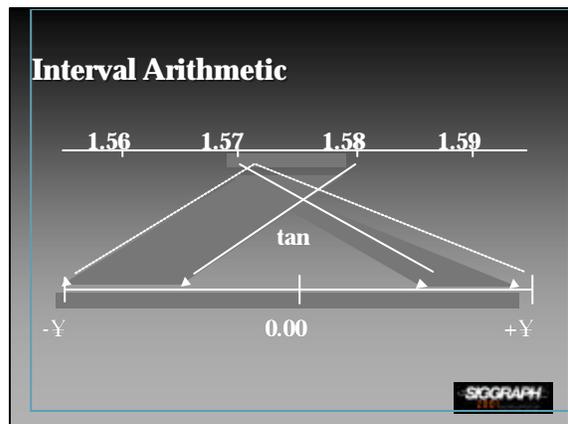
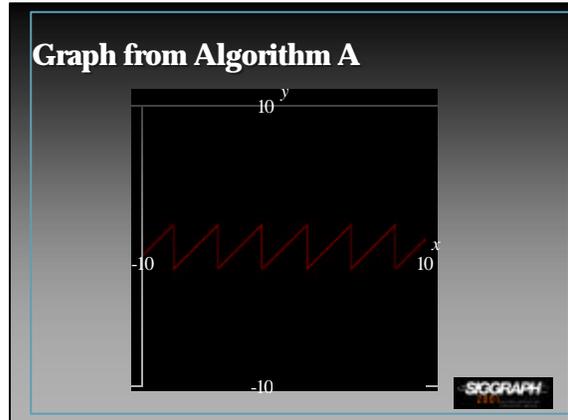
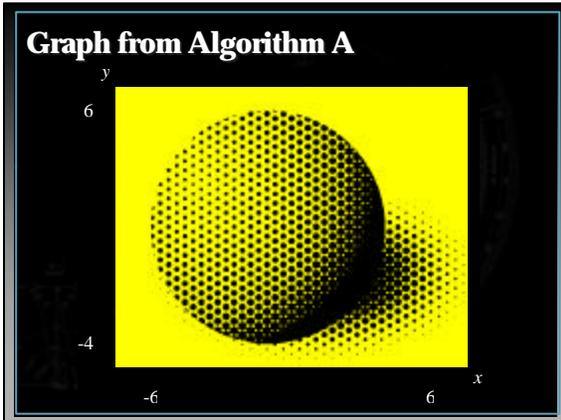


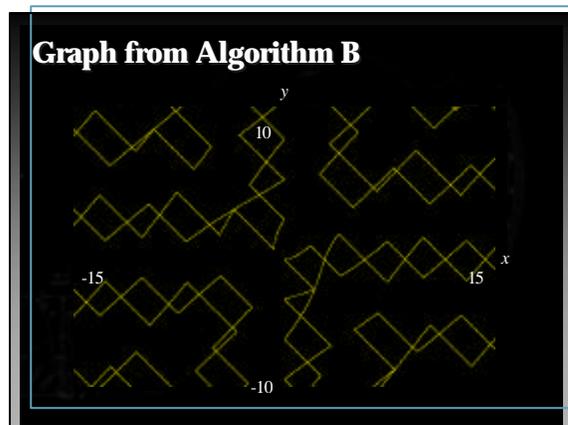
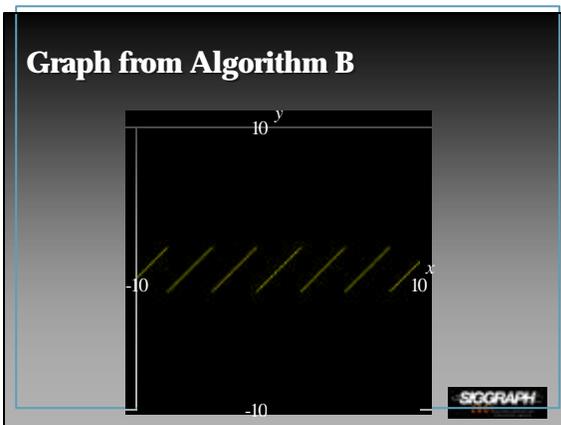
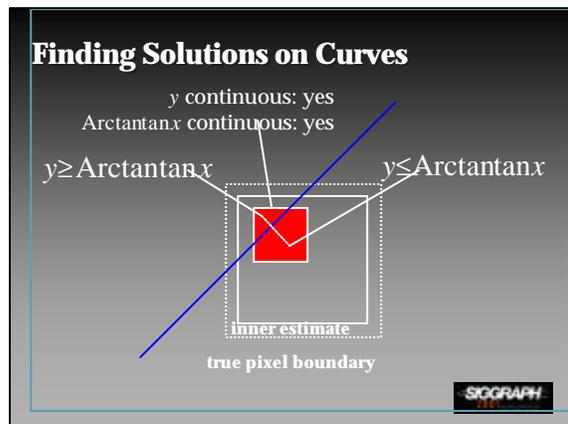
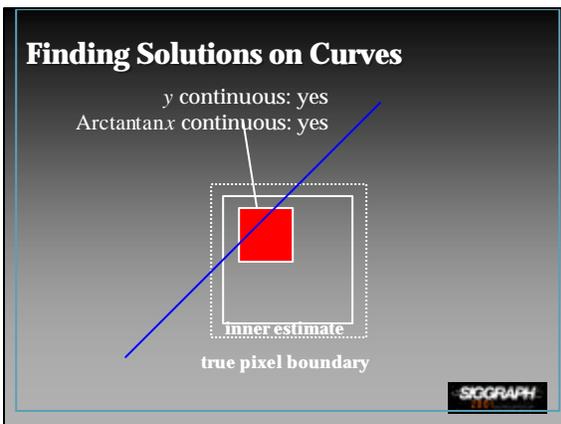
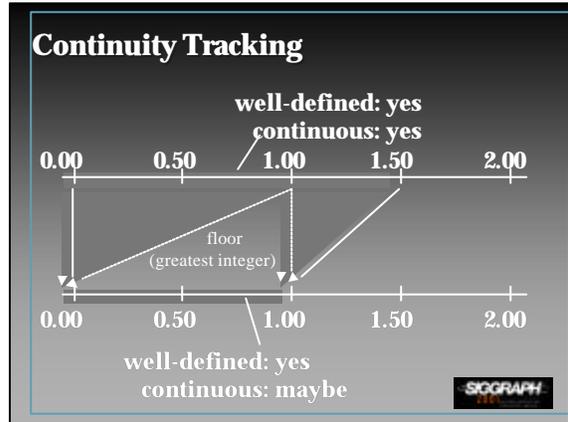
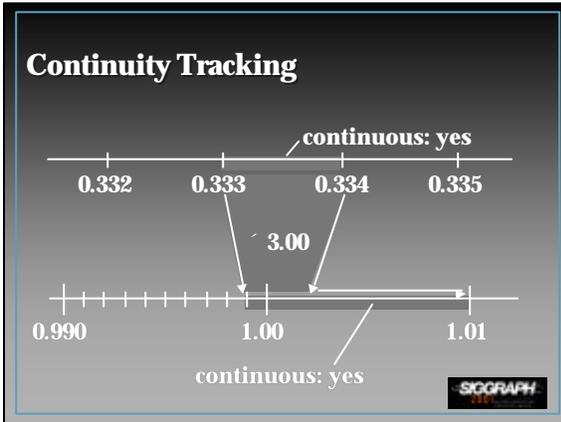
Is $\sqrt{x} < y$? Maybe.

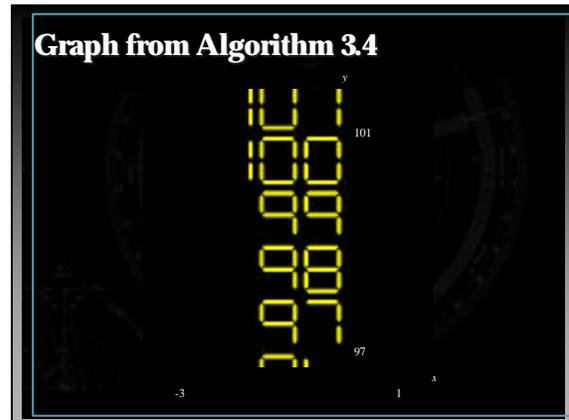
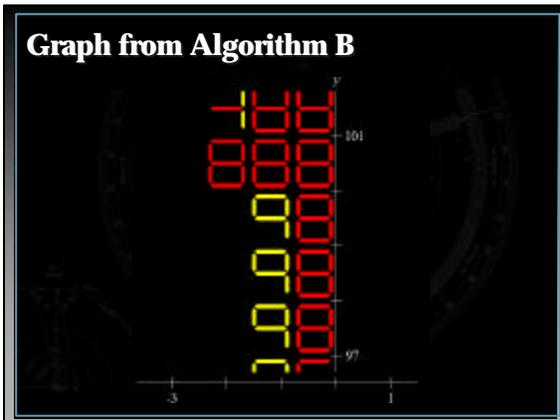
SIGGRAPH











Conclusion

- **Most graphing programs are not reliable**
 - Reliable graphing programs do exist (Grafeq)
- **Red pixels are useful**
- **Be careful when using interval arithmetic**
 - Keeping track of the mathematical properties of evaluated formulae is possible and useful



Future Work

- **Use other colors besides red**
 - Display topological information
- **Tackle a larger class of formulae**
 - integration, differentiation, iteration, ...
- **Animation**
 - visualize role of parameters
- **3D**



Acknowledgements

I would like to thank:

- Alain Fournier;
- my supervisor, Eugene Fiume;
- John Hughes and the other paper reviewers, for their helpful comments.



Contact Information

Jeff Tupper:

- mooncake@dgp.toronto.edu
- www.dgp.toronto.edu/~mooncake

Grafeq:

- www.peda.com/grafeq
- Creative Applications Lab 1PM-2PM Today



