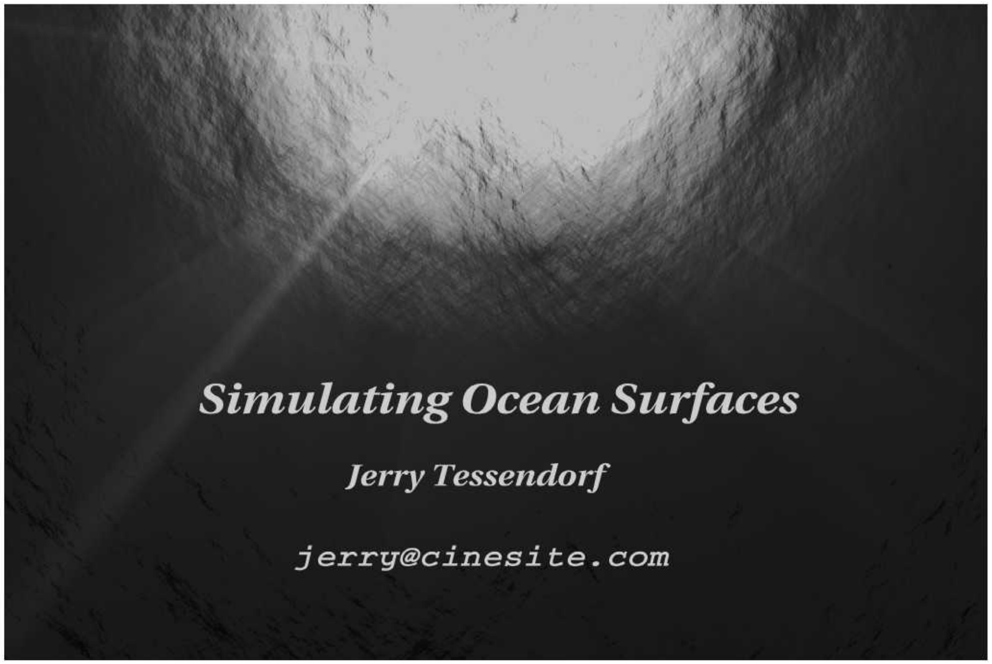
01L



Simulating Ocean Surfaces
Jerry Tessendorf
jerry@cinesite.com

01R

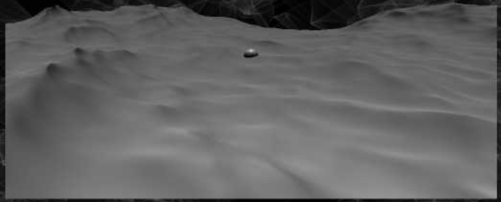
02L

02R

Objectives

- *Introduce Oceanographic concepts & terminology*
- *Present mathematics for generating random waves*
- *Present optical and shader properties of the ocean*
- *Provide hints to achieving realistic look*
- *Demo more advance volume and "dynamic" things*

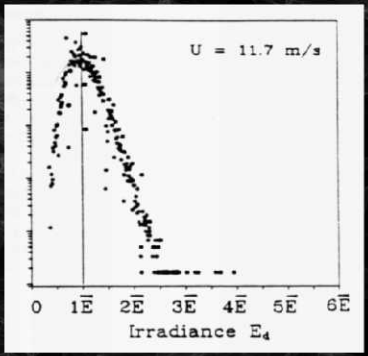
03L



$$\tilde{h}_0(\mathbf{k}) = \frac{1}{\sqrt{2}}\left(\xi_r + i\xi_i\right)\sqrt{P_h(\mathbf{k})}$$

$$\begin{aligned}\tilde{h}(\mathbf{k},t) &= \tilde{h}_0(\mathbf{k})\exp\left\{i\omega(k)t\right\} \\ &+ \tilde{h}_0^*(-\mathbf{k})\exp\left\{-i\omega(k)t\right\}\end{aligned}$$

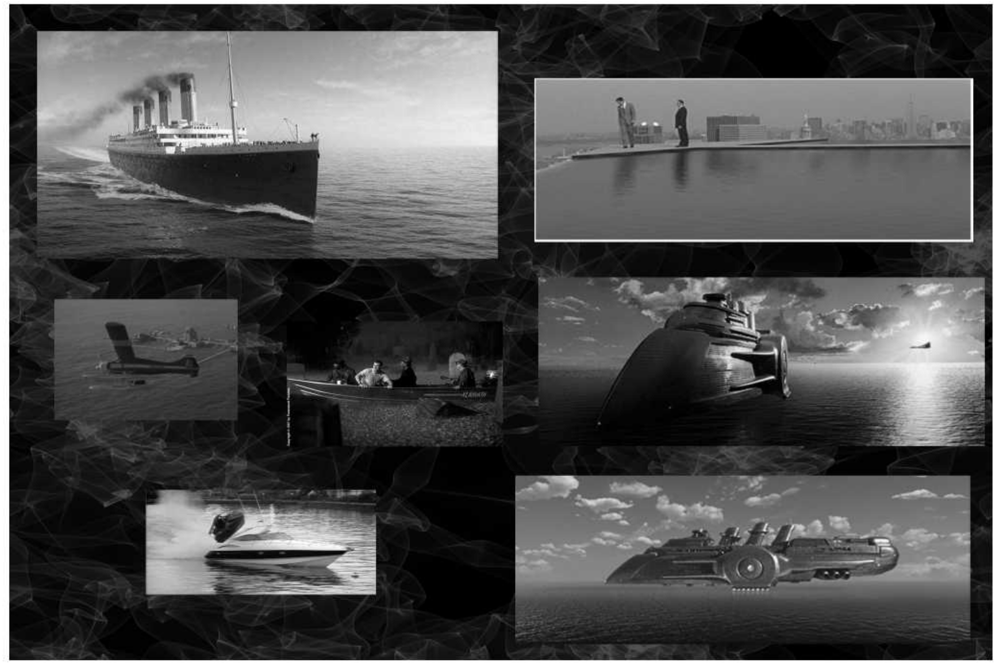
$$h(\mathbf{x},t) = \sum_{\mathbf{k}} \tilde{h}(\mathbf{k},t)\ \exp\left(i\mathbf{k}\cdot\mathbf{x}\right)$$



03R

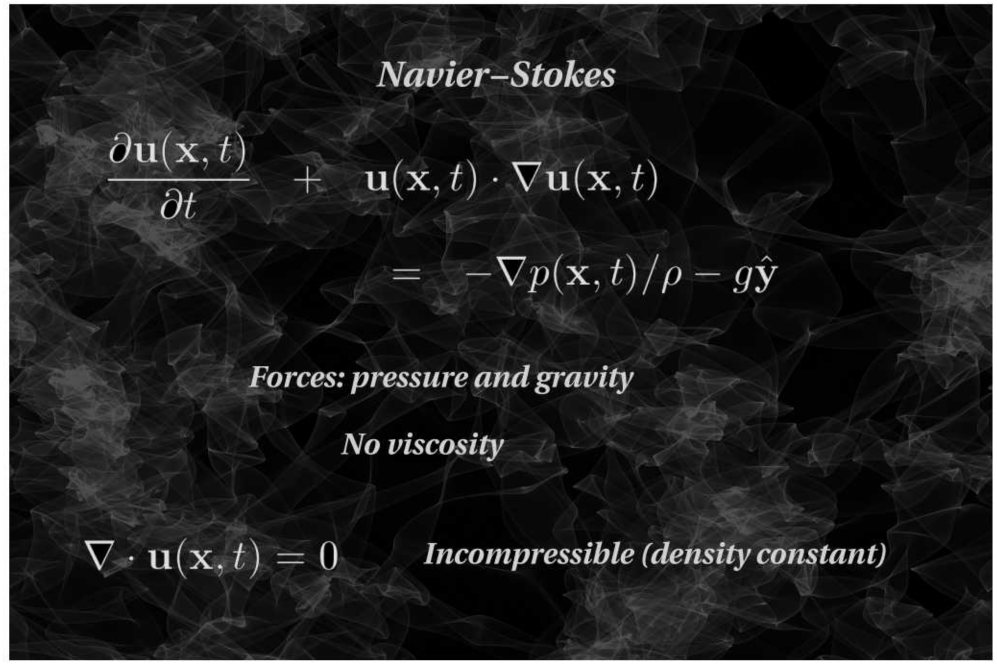

Examples
Waterworld
13th Warrior
Fifth Element
Truman Show
Titanic
Double Jeopardy
Hard Rain
Deep Blue Sea
Devil's Advocate
13 Days
Contact
Virus
20,000 Leagues Under the Sea

04L



04R

05L



Navier–Stokes

$$\frac{\partial \mathbf{u}(\mathbf{x},t)}{\partial t} + \mathbf{u}(\mathbf{x},t) \cdot \nabla \mathbf{u}(\mathbf{x},t)$$

$$= -\nabla p(\mathbf{x},t)/\rho - g\hat{\mathbf{y}}$$

Forces: pressure and gravity

No viscosity

$\nabla \cdot \mathbf{u}(\mathbf{x},t) = 0$ *Incompressible (density constant)*

05R

Potential Flow

$$\mathbf{u}(\mathbf{x},t) = \nabla\phi(\mathbf{x},t)$$

Transforms the Navier Stokes Equations

$$\frac{\partial\phi(\mathbf{x},t)}{\partial t} + \frac{1}{2}\left|\nabla\phi(\mathbf{x},t)\right|^2 + \frac{p(\mathbf{x},t)}{\rho} + g\mathbf{x}\cdot\hat{\mathbf{y}} = 0$$

$$\nabla^2\phi(\mathbf{x},t) = 0$$

06L

06R

In Water Volume

$$\nabla^2 \phi(\mathbf{x}, t) = 0$$

$$\phi(\mathbf{x}) = \int_{\partial V} dA' \left\{ \frac{\partial \phi(\mathbf{x}')}{\partial n} G(\mathbf{x}, \mathbf{x}') - \phi(\mathbf{x}') \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial n} \right\}$$

At the Surface

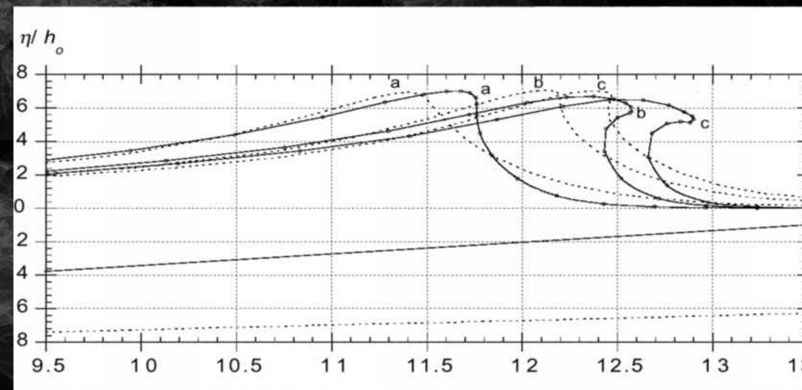
$$\frac{\partial \phi(\mathbf{r}, t)}{\partial t} = -\frac{1}{2} |\nabla \phi(\mathbf{r}, t)|^2 - \frac{p(\mathbf{r}, t)}{\rho} - g\mathbf{r} \cdot \hat{\mathbf{y}}$$

$$\frac{d\mathbf{r}(t)}{dt} = \mathbf{u}(\mathbf{r}, t) = \nabla \phi(\mathbf{r}, t)$$

Dynamics of points on surface

07L

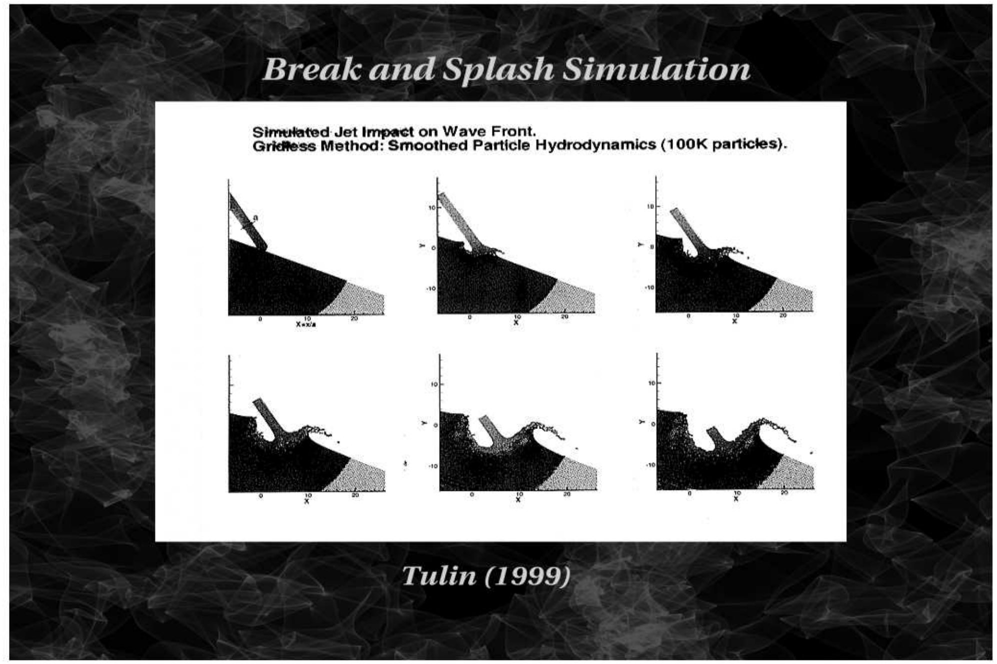
Numerical Wave Tank Simulation



Grilli, Guyenne, and Dias (2000)

07R

08L


Break and Splash Simulation
Simulated Jet Impact on Wave Front.
Gridless Method: Smoothed Particle Hydrodynamics (100K particles).
Tulin (1999)

08R

Simplifying the Problem

Original equation at 3D points in volume

$$\frac{\partial \phi(\mathbf{r},t)}{\partial t} = -\frac{1}{2}\left|\nabla \phi(\mathbf{r},t)\right|^2 - \frac{p(\mathbf{r},t)}{\rho} - g\mathbf{r}\cdot\hat{\mathbf{y}}$$

Reduced equation at 2D points (x, z) on surface

$$\frac{\partial \phi(x,z,t)}{\partial t} = -gh(x,z,t)$$

09L

09R

Simplify Surface Dynamics

Vertical velocity related to height

$$\frac{\partial h(x,z,t)}{\partial t} = \hat{\mathbf{y}} \cdot \nabla \phi(x,z,t)$$

Use incompressibility to achieve surface–only prescription

$$\left(\hat{\mathbf{y}} \cdot \nabla\right) \phi \sim \left(\sqrt{-\nabla_H^2}\right) \phi = \left(\sqrt{-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2}}\right) \phi$$

11L

Linearized Surface Waves

$$\frac{\partial h(x,z,t)}{\partial t} = \sqrt{-\nabla_H^2}\,\phi(x,z,t)$$

$$\frac{\partial \phi(x,z,t)}{\partial t} = -g h(x,z,t)$$

General solution easily achieved in terms of the Fourier Transform

11R

Solution of Linearized Wave Equations

The general solution in terms of Fourier Transform is

$$h(x, z, t) = \int_{-\infty}^{\infty} dk_x \; dk_z \; \tilde{h}(\mathbf{k}, t) \exp \{ i(k_x x + k_z z) \}$$

with a time-dependent amplitude that depends on the dispersion relationship

$$\omega_0(\mathbf{k}) = \sqrt{g \, |\mathbf{k}|}$$

$$\tilde{h}(\mathbf{k}, t) = \tilde{h}_0(\mathbf{k}) \exp \{ -i\omega_0(\mathbf{k}) t \} + \tilde{h}_0^*(-\mathbf{k}) \exp \{ i\omega_0(\mathbf{k}) t \}$$

Oceanography

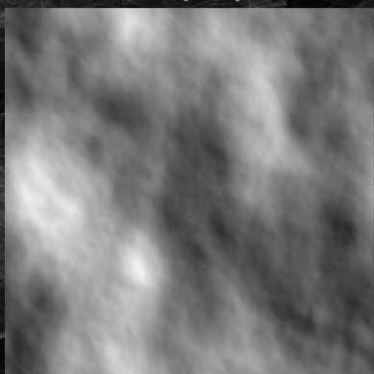
- Decades of detailed measurements support a statistical description of waves.
- The wave height has a statistical spectrum

$$\left\langle \left| \tilde{h}_0(\mathbf{k}) \right|^2 \right\rangle \equiv P_0(\mathbf{k})$$

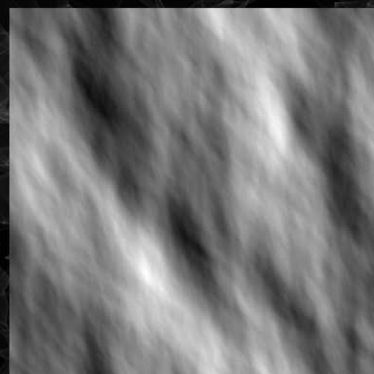
- Oceanographic models give P_0 in terms of wind velocity and other parameters.

Variation in Wave Height Field

Pure Phillips Spectrum



Modified Phillips Spectrum



14L

Model Spectra

Phillips spectrum

$$L = V^2/g$$

$$P_0(\mathbf{k}) = \frac{\exp\left(-1/k^2L^2\right)}{k^4}|\hat{\mathbf{k}}\cdot\hat{\mathbf{V}}|^2$$

JONSWAP Frequency spectrum

$$P_0(\omega) = P_0\omega^{-5}\exp\left\{-\frac{5}{4}\left(\frac{\omega}{\Omega}\right)^{-4} + e^{-(\omega-\Omega)^2/2(\sigma\Omega)^2}\ln\gamma\right\}$$

14R

Computational Data

$$\tilde{h}_0(\mathbf{k}) = \xi \, e^{i\theta} \sqrt{P_0(\mathbf{k})}$$

$$\mathbf{k} = (k_x, k_z)$$

$$k_x = \frac{2\pi}{\Delta x}\frac{n}{N} \quad (n = -N/2, \ldots, (N-1)/2)$$

$$k_z = \frac{2\pi}{\Delta z}\frac{m}{M} \quad (m = -M/2, \ldots, (M-1)/2)$$

ξ ***Gaussian Random Number w/ mean 0 & std dev 1***

θ ***Uniform Random Number [0,2π]***

15L

FFT

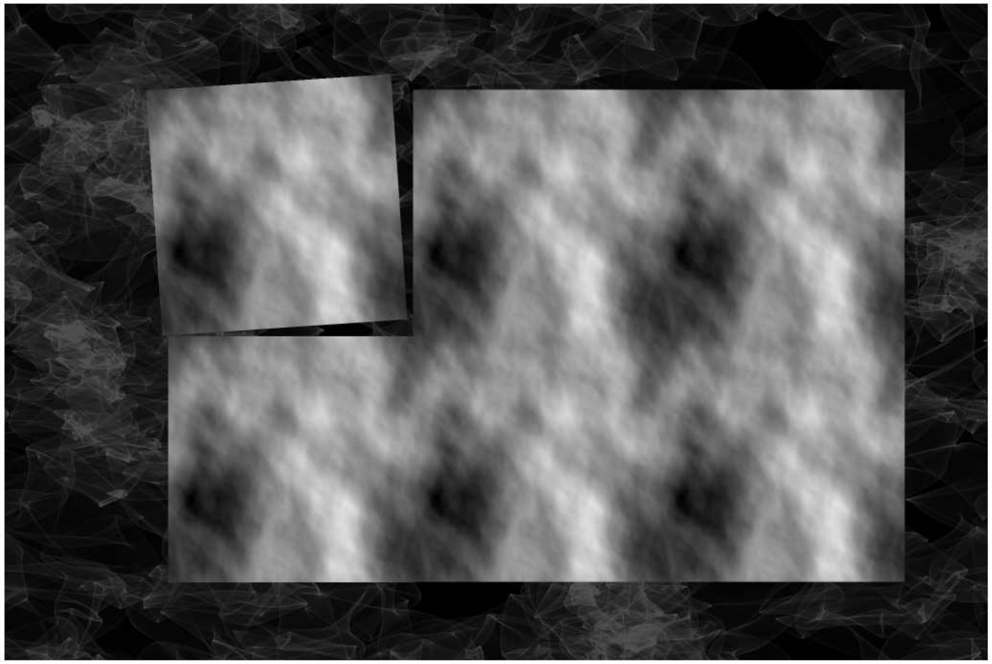
Use Fast Fourier Transform to compute $h(\mathbf{x}, t)$ ***on the grid***

$$x = n\Delta x$$

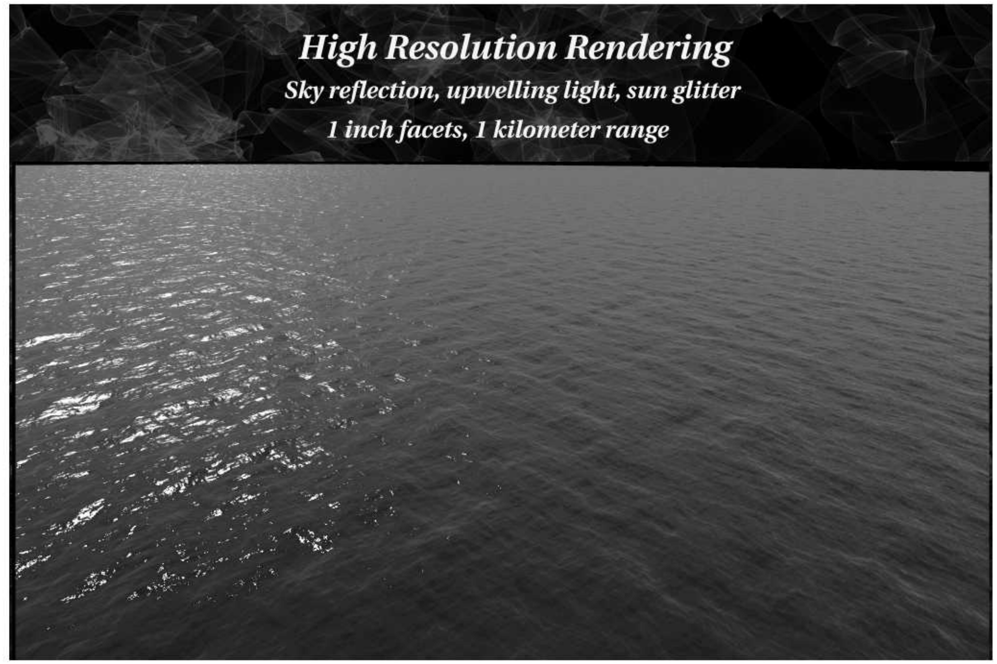
$$z = m\Delta z$$

The outcome is periodic and tiles seamlessly

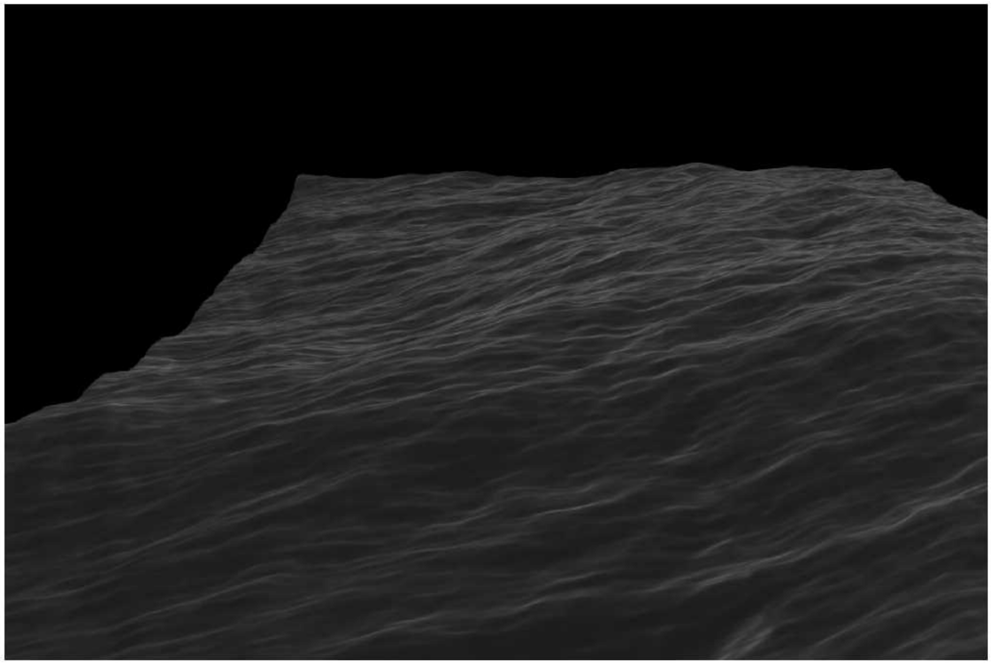
15R



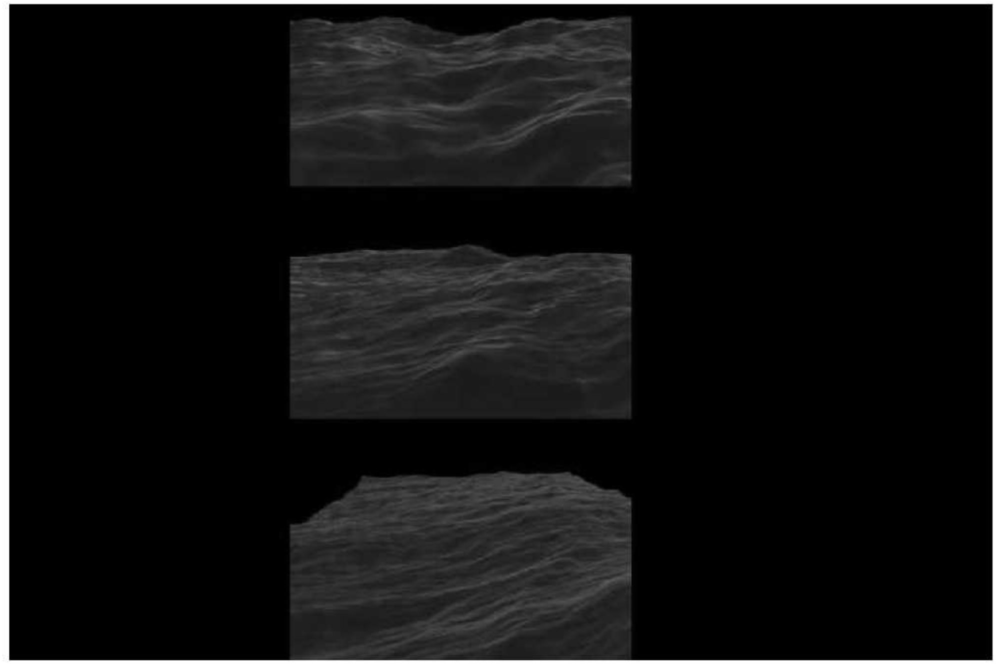
16L


High Resolution Rendering
Sky reflection, upwelling light, sun glitter
1 inch facets, 1 kilometer range

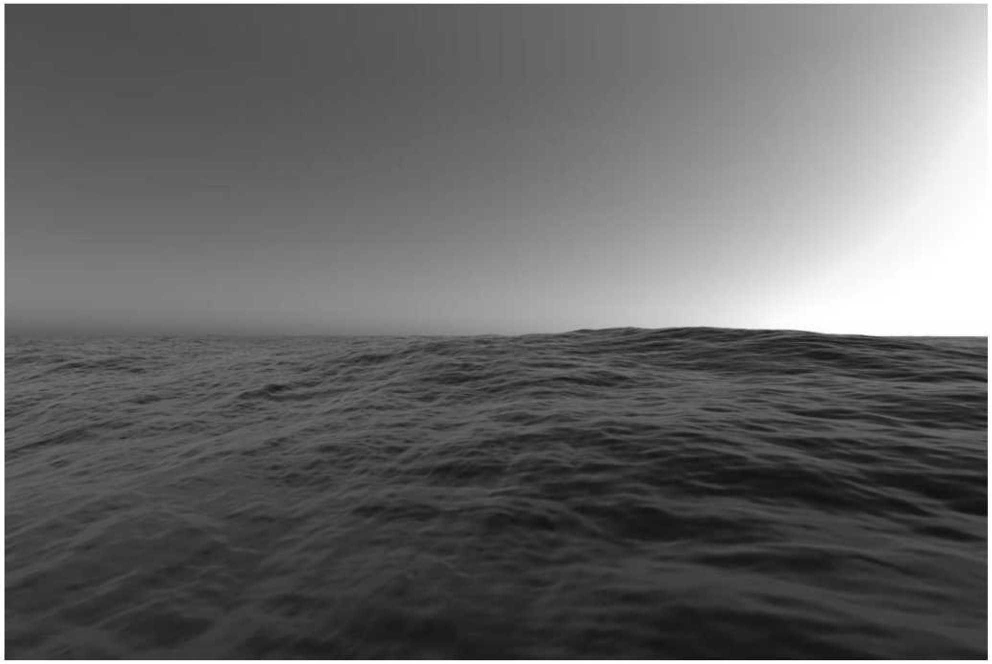
16R



17L

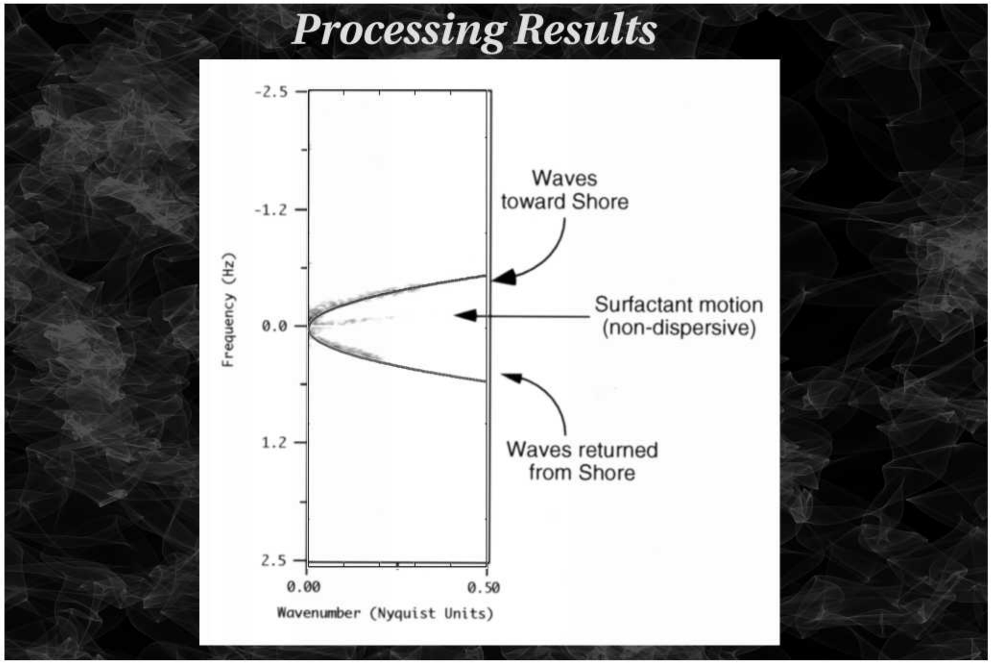


17R

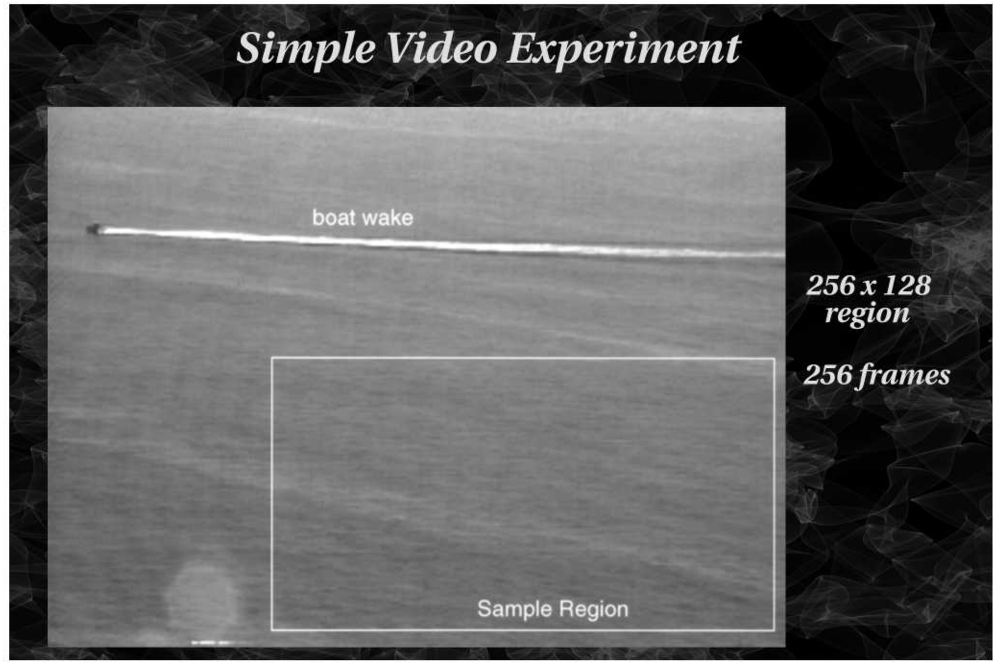


18L

18R


Processing Results
-2.5
-1.2
0.0
1.2
2.5
Frequency (Hz)
0.00
0.50
Wavenumber (Nyquist Units)
Waves
toward Shore
Surfactant motion
(non-dispersive)
Waves returned
from Shore

19L


Simple Video Experiment
boat wake
256 x 128
region
256 frames
Sample Region

19R

Choppy/Cuspy Near–Breaking Waves

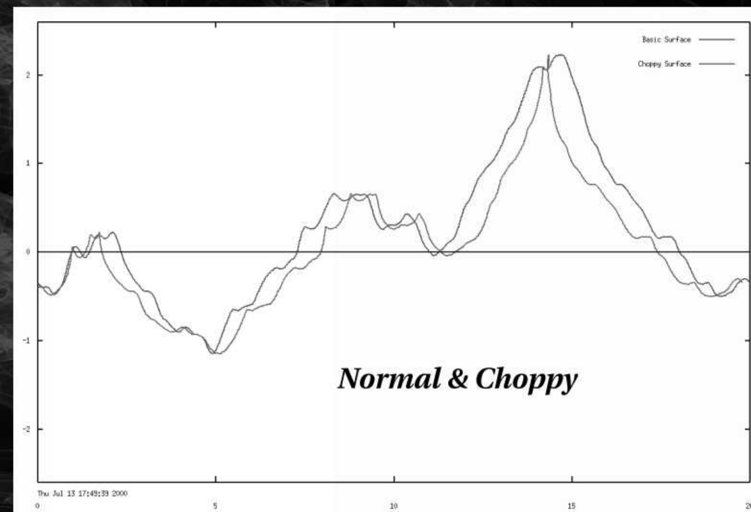
Horizontal velocity moves pieces of wave around

Wave at $\mathbf{x} = (x, z)$ ***morphs horizontally to*** $\mathbf{x} + \lambda \mathbf{D}(\mathbf{x}, t)$

$$\mathbf{D}(\mathbf{x}, t) = -i \int d^2k \, \frac{\mathbf{k}}{|\mathbf{k}|} \, \tilde{h}(\mathbf{k}, t) \exp\left\{ i(k_x x + k_z z) \right\}$$

The factor λ ***allows artistic control over the magnitude of the morph***

20L



20R

Cuspy Waves: Detecting Overlap

The transformation $\mathbf{x} \mapsto \mathbf{X}(\mathbf{x}, t) = \mathbf{x} + \lambda \mathbf{D}(\mathbf{x}, t)$ is unique and invertible as long as the surface does not intersect itself.

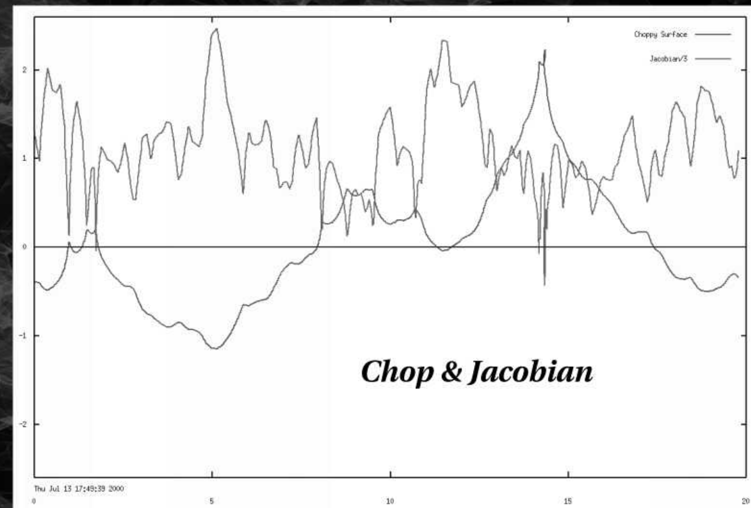
When the mapping intersects itself, it is not unique. The quantitative measure of this is the Jacobian Matrix

$$J(\mathbf{x}, t) = \left[\begin{array}{cc} \partial \mathbf{X}_x / \partial x & \partial \mathbf{X}_x / \partial z \\ \partial \mathbf{X}_z / \partial x & \partial \mathbf{X}_z / \partial z \end{array} \right]$$

with this Jacobian, the sign of the surface intersecting itself is

$$det(J) \leq 0$$

21L



21R

Learning More about Overlap

J has two eigenvalues $J_1 \leq J_2$, and two eigenvectors $\hat{\mathrm{e}}_1$ and $\hat{\mathrm{e}}_2$.

$$J = J_1 \hat{\mathrm{e}}_1 \hat{\mathrm{e}}_1 + J_2 \hat{\mathrm{e}}_2 \hat{\mathrm{e}}_2$$

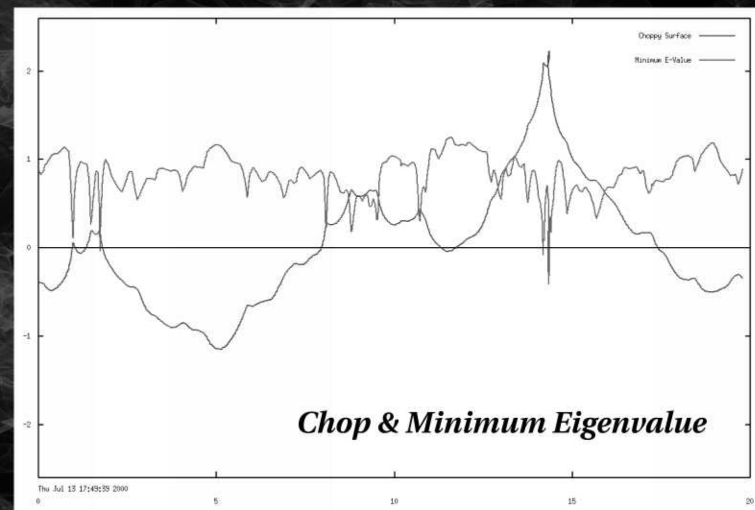
and

$$det(J) = J_1 J_2$$

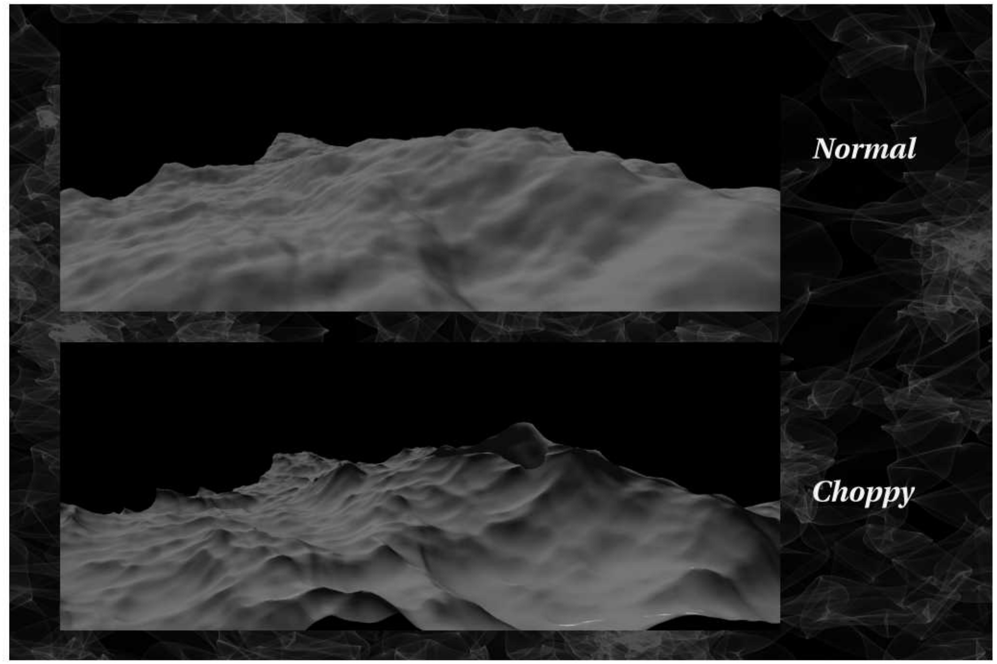
With no displacement, $J_1 = J_2 = 1$. As displacement leads to overlap, J_2 stays positive while J_1 becomes negative.

In overlap, with $J_1 < 0$, $\hat{\mathrm{e}}_1$ is the alignment direction of the overlap.

22L

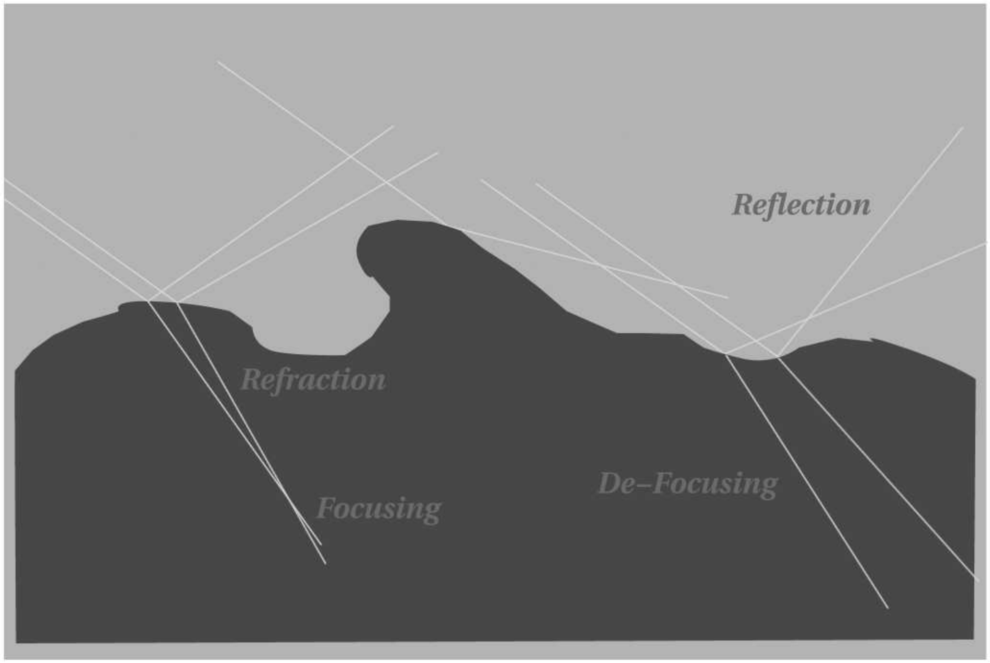


22R



23L

23R



24L

Water Optics

Interface

Very good specular reflector

Most light is refracted into volume

Surface roughness is very important

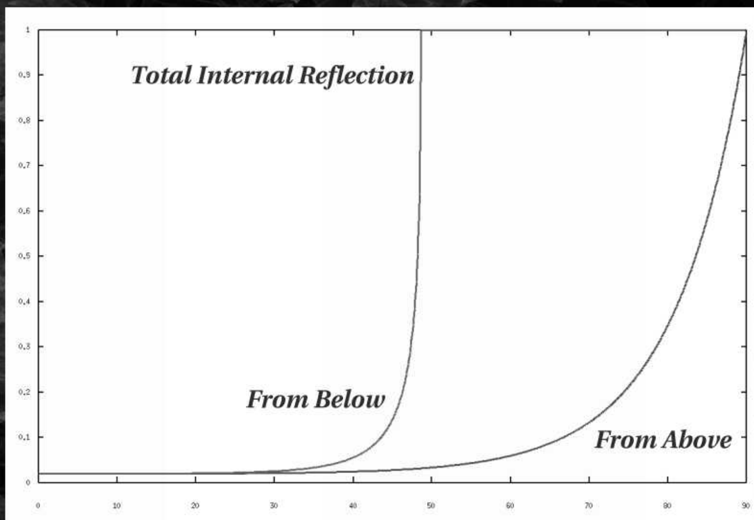
Underwater Volume

Downwelling sunlight scattered in all directions

Scatterers: one–celled organisms, dead matter, dirt, others

24R

Water Surface Reflectivity



Incidence Angle (degrees)

25L

Specular Reflection & Transmission

Sky light reflected by surface to camera

$$L_{Specular} = R_s L_{Sky}\left(\hat{\mathbf{n}}_{Specular}\right)$$

$$\hat{\mathbf{n}}_{Specular} = \hat{\mathbf{n}}_{View} - 2\cos\left(\theta_{View}\right)\hat{\mathbf{n}}_{Surface}$$

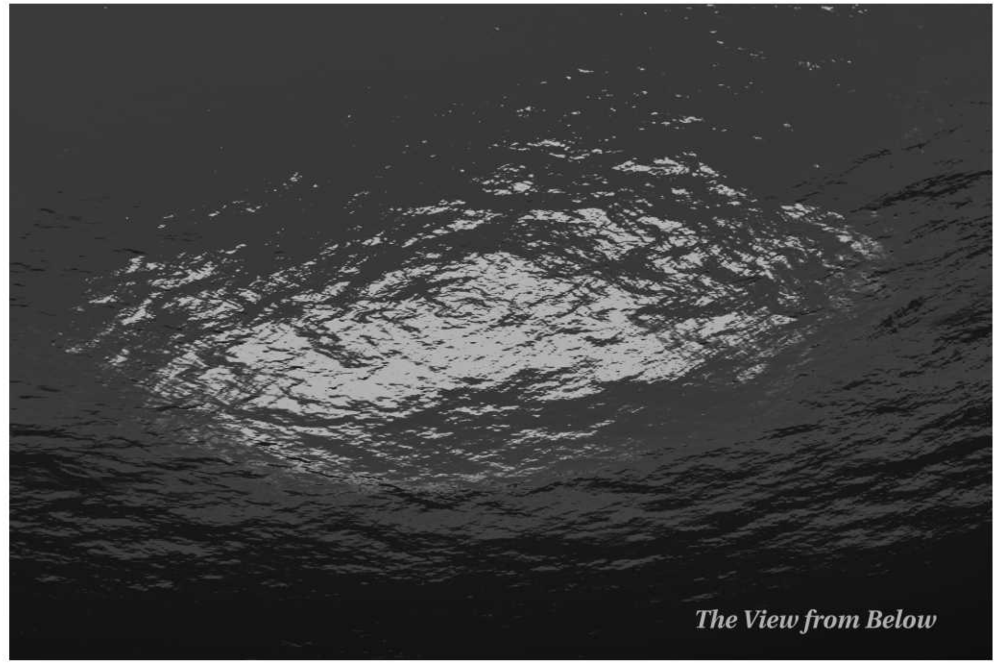
Surface Reflectivity

$$R_s = \frac{1}{2}\left\{\frac{\sin^2(\theta_{View} - \theta_{Transmit})}{\sin^2(\theta_{View} + \theta_{Transmit})} + \frac{\tan^2(\theta_{View} - \theta_{Transmit})}{\tan^2(\theta_{View} + \theta_{Transmit})}\right\}$$

$$\sin\left(\theta_{Transmit}\right) = \frac{n_a}{n_w}\sin\left(\theta_{View}\right) \qquad n_a/n_w = 0.75$$

25R

26L

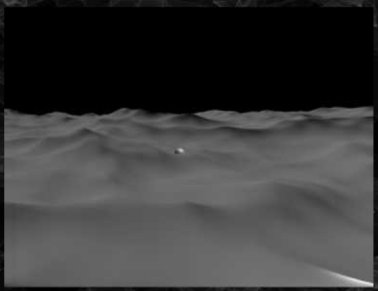


26R

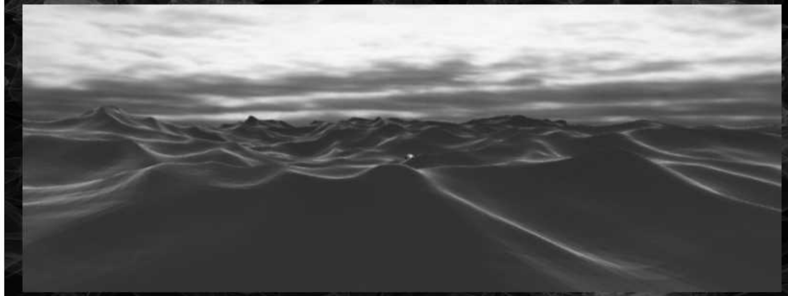
```
surface watercolorshader(
    color upwelling = color(0, 0.2, 0.3);
    color sky       = color(0.69,0.84,1);
    color air       = color(0.1,0.1,0.1);
    float nSnell    = 1.34;
    float Kdiffuse  = 0.91;
    string envmap   = "";
                        )
{
  float reflectivity;
  vector nI = normalize(I);
  vector nN = normalize(Ng);
  float costhetai = abs(nI . nN);
  float thetai = acos(costhetai);
  float sinthetat = sin(thetai)/nSnell;
  float thetat = asin(sinthetat);
  if(thetai == 0.0)
  {
    reflectivity = (nSnell - 1)/(nSnell + 1);
    reflectivity = reflectivity * reflectivity;
  }
  else
  {
    float fs = sin(thetat - thetai)/sin(thetat + thetai);
    float ts = tan(thetat - thetai)/tan(thetat + thetai);
    reflectivity = 0.5 * ( fs*fs + ts*ts );
  }
  vector dPE = P-E;
  float dist = length(dPE) * Kdiffuse;
  dist = exp(-dist);
  if(envmap != ""){ sky = color environment(envmap, nN);}
  Ci =  dist * ( reflectivity * sky  + (1-reflectivity) * upwelling )
     + (1-dist)* air;
}
```

Simple Surface Shader

27L



Plastic Shader



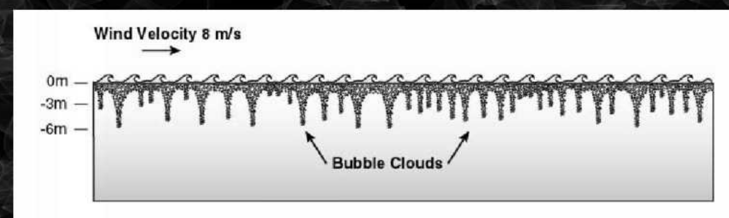
Water Shader

27R

Light Scattering by Water Volume

Strong color spectrum dependence

- Water molecules are relatively small contributor
- Single–celled organisms
- Waste and dead matter
- Dirt, river runoff
- Microbubbles from wave action (new research)

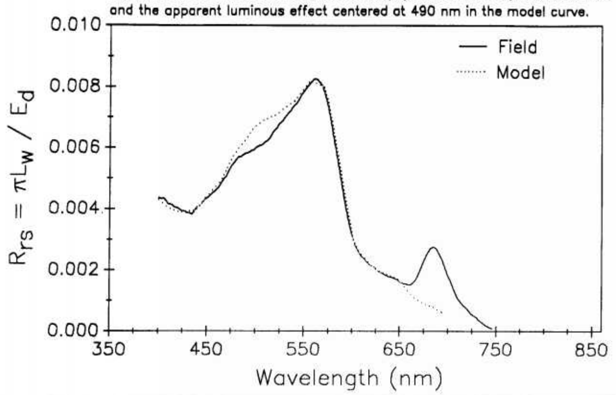


28L

28R

Bulk Reflectivity Measurements

FIG. 4: Field vs model curves of $R_{rs}(\lambda)$ for Station 55, a close inshore station south of Pt. Arena. Note the Chlorophyll fluorescence peak at 685 nm and the apparent luminous effect centered at 490 nm in the model curve.



Peacock et al (1990)

29L

Reflection by Water Volume

Skylight transmitted through surface, scatters in the volume, and some is transmitted back out through the surface

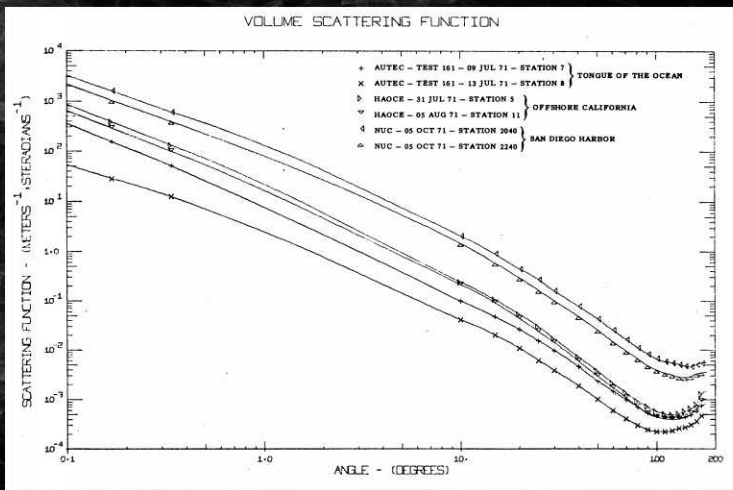
$$L_{Upwelling} = T_{down} \frac{E_{sky}}{\pi} R_{Volume}$$

The "bulk" reflectivity R_{Volume} accounts for complex scattering and attenuation by the volume.

Experimentally, R_{Volume} is color–biased toward blue–green and green, depending on water clarity

29R

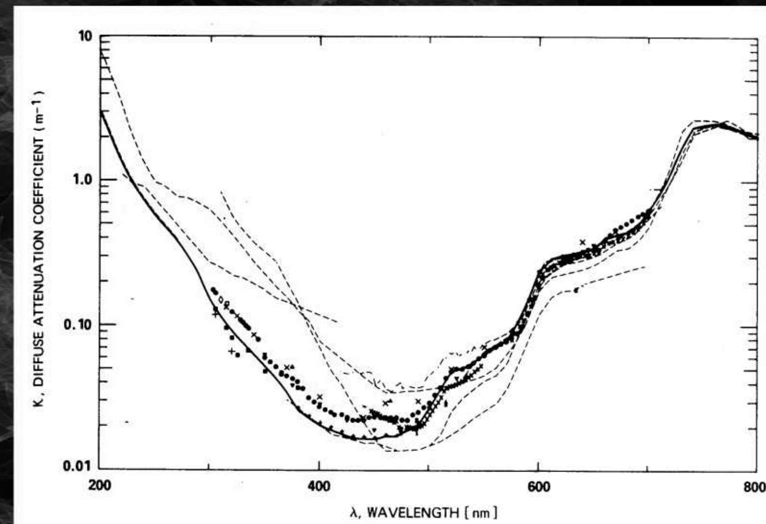
Scattering Function for Water



Petzhold (1972)

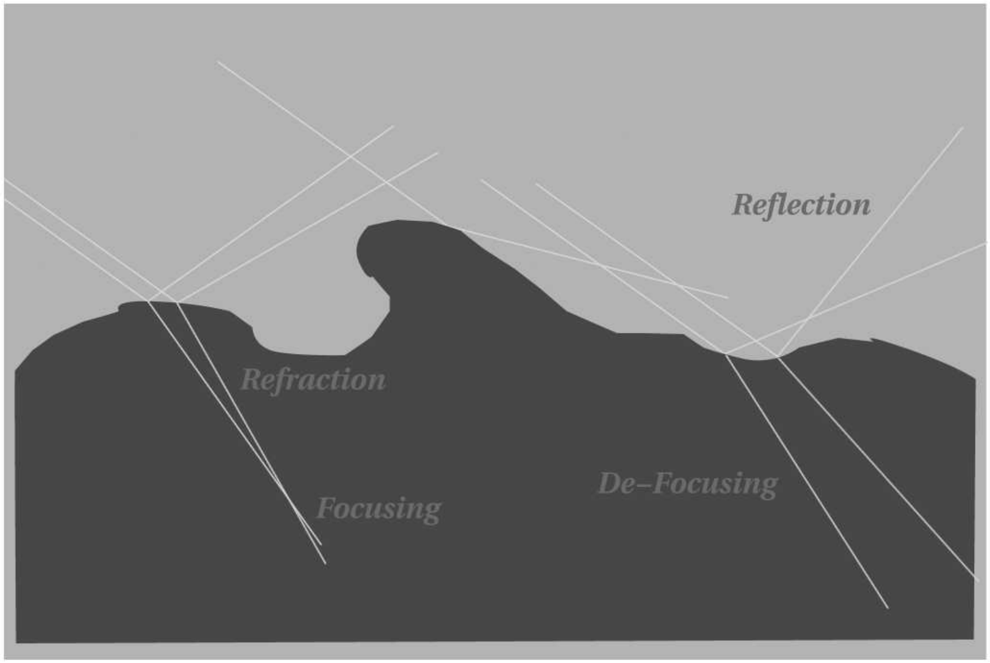
30L

Spectral Diffuse Attenuation



Smith & Baker (1981)

30R



31L

Volume Effects of a Wavy Surface

The wavy rough surface acts as a lense on the light that passes through. Light is focussed by peaks, defocused by troughs.

Caustics Patterns of light and dark on a surface underwater, which evolve in complex ways.

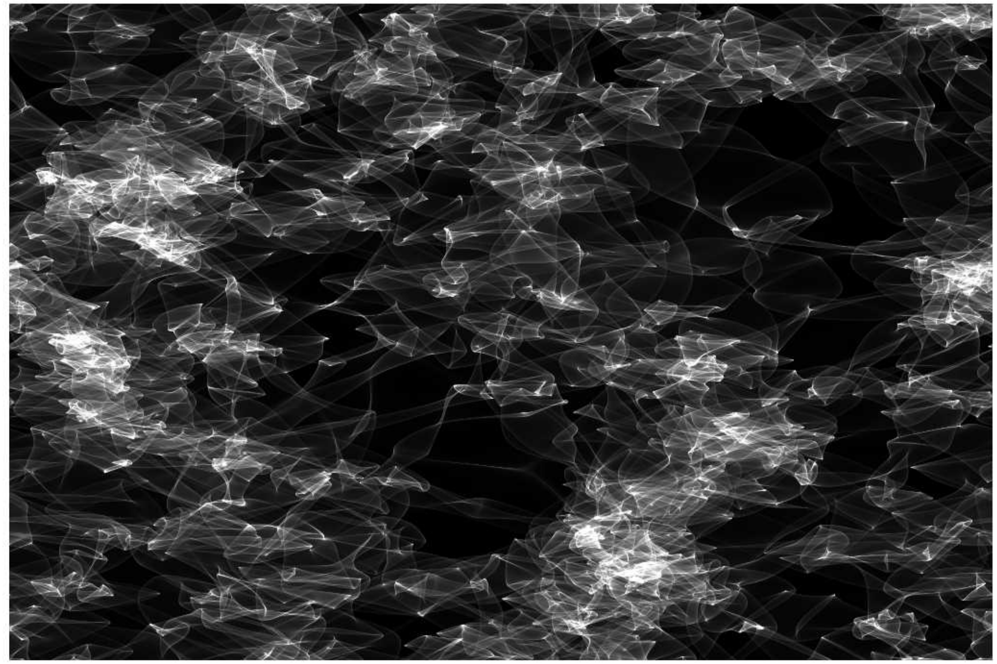
Sunbeams Shafts of light that swing around in the volume, extending from the surface and decaying with depth.

31R

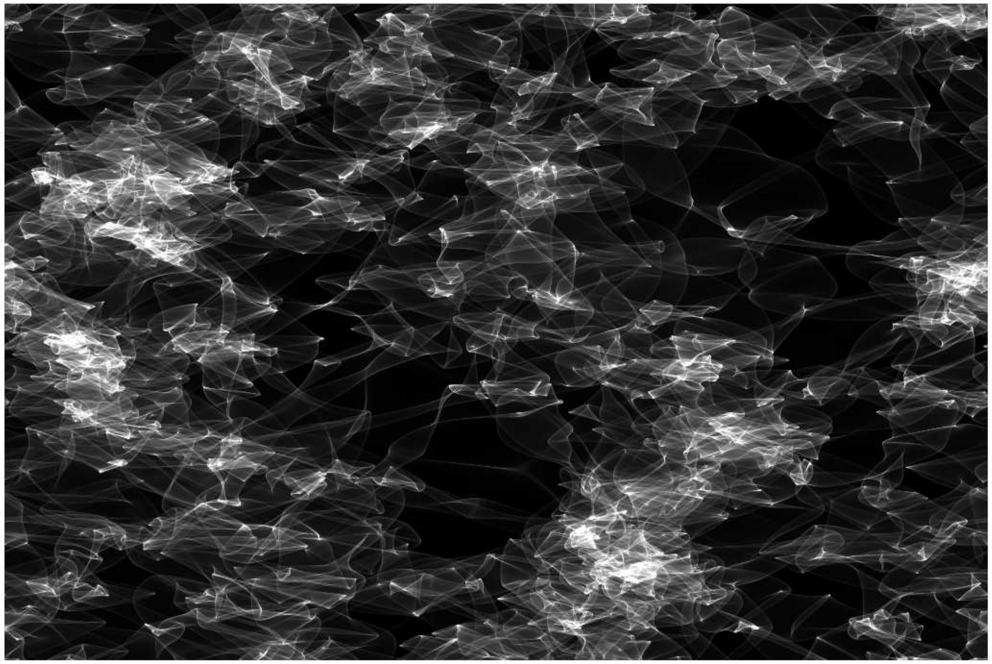
Caustics on a flat submerged surface

- Direct sunlight is refracted by the wavy ocean surface
- Wave peaks focus light, wave valleys defocus light
- After a wave-dependent depth, all light is defocused
- Constant flux: shaft area $\times$ light intensity = constant
- Caustic pattern persists deep because of accumulation from many waves.

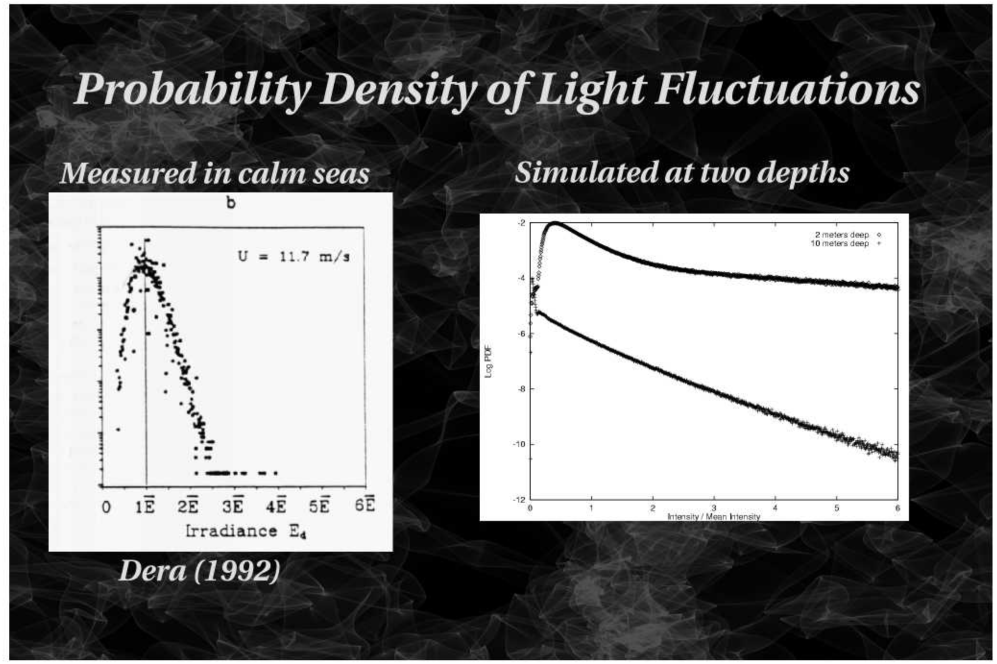
32L



32R



33L

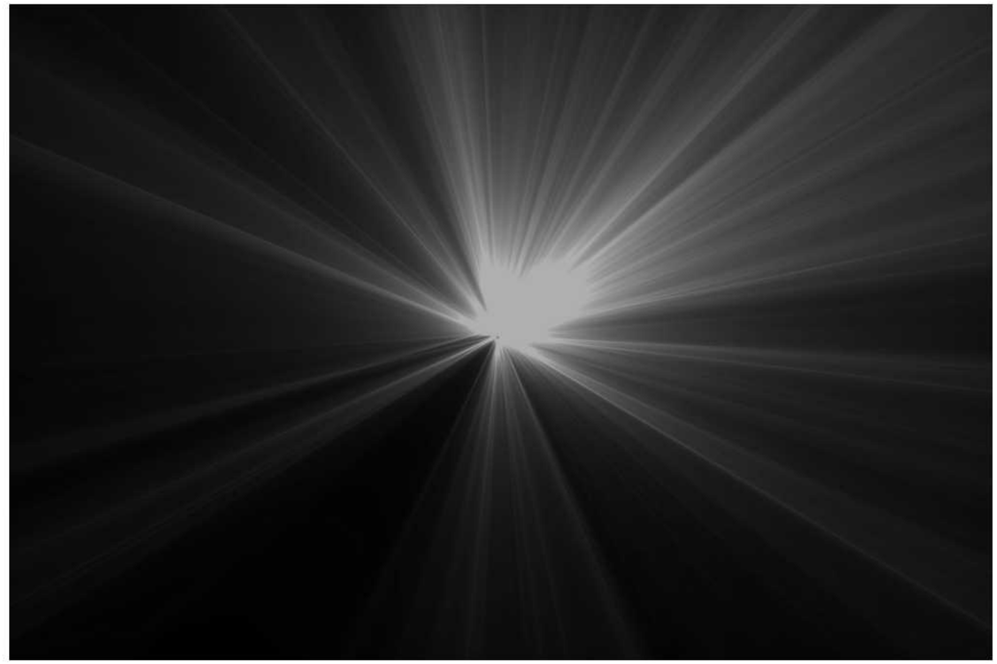

Probability Density of Light Fluctuations
Measured in calm seas
Simulated at two depths
b
U = 11.7 m/s
0 1Ē 2Ē 3Ē 4Ē 5Ē 6Ē
Irradiance E4
Dera (1992)
2 meters deep
10 meters deep
Log PDF
Intensity / Mean Intensity

33R

Underwater Sunbeams

- Due to refraction of sunlight by wavy surface, like caustics
- Light scattered by water volume within a light shaft is viewed by the camera
- Water volume scatter very dependent on water quality, and has a strong angular variation

34L



34R

Where to go from here

- Spray and foam physically related to wave motion
- Potential Flow calculations
- Breaking waves and beaches with high fidelity
- Coupling to other fluid dynamics (objects in water, wind gusts, ...)
- More complex environments with radiosity.

35L

35R